Do Your Learners Do the Thinking?

DEVELOPING LEARNERS'
HIGHER-ORDER THINKING SKILLS
IN MATHEMATICS FOR GRADES 7, 8, 9 & 10

A TEACHER'S RESOURCE

A Professional Learning Package on the
PPST Indicators 1.5.2 and 1.5.3

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TABLE OF CONTENTS

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INTRODUCTION TO TEACHER'S RESOURCE

Hello, dear Teacher! Welcome to this
Professional Learning Package in Mathematics!

The **P**rofessional **L**earning **P**ackage (PLP) in Mathematics is composed of:

- ✓ **this Teacher's Resource in Mathematics, which includes 24 assessment items involving 93 individual Questions;**
- ✓ a Mentor's Guide.

This *Teacher's Resource* is designed to set up instructional support for you to implement teaching strategies effectively in helping learners develop higher-order thinking skills through the PPST Indicator 1.5.2 and Indicator 1.5.3; thus, responding to the demands of the **P**rogramme for **I**nternational **S**tudent **A**ssessment (PISA) and other international and national assessments.

Developing **H**igher-**O**rder **T**hinking **S**kills (**HOTS**) among learners is crucial to prepare them to face and manage the challenges of the 21st-century academic and social environment as well as to help them reach their full potential.

Through this learning package, you will be able to determine and obtain essential methods and resources for equipping learners with lifelong critical-thinking competencies. Likewise, this learning package will be relevant to your professional practice in enhancing your knowledge and skills in the identified PPST indicators.

Structure of the Teacher's Resource

This Teacher's Resource contains twenty-four (24) SOLO-based assessment items involving ninety-three (93) individual Questions

- eight (8) Items involving thirty-four (34) individual Questions for Grade 7;
- two (2) Items involving seven (7) individual Questions for Grade 8;
- eight (8) Items involving twenty-eight (28) individual Questions for Grade 9; and
- six (6) Items involving twenty-four (24) individual Questions for Grade 10.

In addition, acceptable answers for each question/activity can be found in a separate section in this resource. Writer's Reflections are included for two main purposes:

1. To assist participants know and understand the intention of the item writers in developing the items – this is important both in understanding the mathematics content being assessed and the strategies being employed to engage students in higher-order thinking.
2. To model the thinking and design processes to consider when developing their own HOTS assessment items.

A valuable approach for ongoing professional learning is for teachers to try to predict how their students are likely to engage with and answer questions, then check after using the assessment items if their predictions are supported. This reflective approach efficiently reinforces professional learning and teachers' skills in designing high quality assessment items.

Further, the SOLO-based assessment items address competencies and topics across the following grade levels: Grades 7, 8, 9, and 10. You may also adjust the complexity of the items to better suit the levels of knowledge, understanding and skills of your students. You are also encouraged to adapt the types of strategies for use with other Grade levels and curriculum content.

Ideas for Mentors:

Using the items in LACs might include:

- Initially, selecting a single Item/or Item set to use to introduce the structure and features of the package to mentees
- Suggesting as small set of items around a single topic for mentees to try before a next LAC session, so the whole group can feedback and share ideas on the same set of items
- Discussing the full range of HOTS strategies being modelled in the resource.
- Asking mentees to identify an item of interest to review, try with their students, evaluate and then report back to their group
- Leading your LACs group to identify some general strategies that support HOTS, e.g., *building questions up in a topic from unistructural to multistructural to relational*; or *using open questions and scaffolds*.

Ideas for Mentees:

Using the items:

- Trying some items themselves to reflect on their science knowledge and understanding
- Trying some item with their students – maybe some in early grades and some in higher grades
- Sharing some items with their school colleagues – this could be useful for collaborative discussions; clarifying the focus the school has on the Philippine Curriculum that they are currently implementing
- Adapting or developing some items for your school's context.

Further or complementary study:

- Reading more about HOTS
- Exploring PISA competencies and PISA testing and how they might be used in your school

OVERVIEW

Programme for International Student Assessment (PISA) and the K to 12 Framework

The Programme for International Student Assessment (PISA), which began in 2000, is an international large-scale assessment (ILSA) by the Organization for the Economic Co-operation and Development (OECD). PISA measures 15-year-olds' ability to use the knowledge and skills they learned in school to real-life situations. PISA does not assess how well learners remember facts but how they are able to interpret texts, solve mathematics problems, or explain phenomena scientifically using their knowledge and reasoning skills. These skills are higher-order thinking skills (HOTS).

The Philippines took part in the PISA international program in 2018. The assessment results, informed as well by findings from other international and national assessments, prompted a more aggressive reform initiative to strengthen teachers' subject knowledge and pedagogy to help improve learner performance. To assist in this reform initiative, the National Educators Academy of the Philippines (NEAP) and the Research Center for Teacher Quality (RCTQ) collaborated to develop Professional Learning Packages (PLPs) in Mathematics, Science, and English/Reading for teachers in Grades 7, 8, 9 and 10.

The professional learning packages in this resource are situated within the context of the Philippines K to 12 Mathematics Curriculum Framework. The curriculum framework is founded on the belief that "mathematics is a skills subject" focused on quantities, shapes and figures, functions, logic, and reasoning. At the heart of the Philippine mathematics curriculum is the need to develop critical-thinking and problem-solving skills, which must be comprehensively and thoroughly learned by the learners (Department of Education, 2016).

PISA and its tests are one specific way to explore the degree to which learners in the Philippines have developed critical thinking and the ability to solve problems. At the same time PISA offers participating nations the opportunity to situate their students' learning within an international context of over 70 countries.

The **S**tructure of the **O**bserved **L**earning **O**utcome (SOLO) model offers a framework, that can act as a link between the Philippine Curriculum, what should be happening in the classroom and the development of PISA competencies evidenced by results of 15 year-olds on PISA tests. Designed initially for use by schools in describing the quality of learner responses, SOLO provides a language to describe the structure of a response that is sufficiently generic to be relevant to all learning situations.

The Philippine Professional Standards for Teachers (PPST) and Higher-Order Thinking Skills (HOTS)

The enhancement of higher-order thinking skills is crucial and imperative in achieving an improved learner performance. To help our teachers, the DepEd sought the issuance of the Philippine Professional Standards for Teachers (PPST) to define what teachers should know, be able to do, and value to achieve competence.

The first Domain of the PPST is **Domain 1: Content Knowledge and Pedagogy**, which focuses on the teachers' ability to apply developmentally appropriate and meaningful pedagogy grounded on content knowledge and current research to promote high-quality learning outcomes.

Under this domain, Proficient (1.5.2) and Highly Proficient (1.5.3) indicators aim to equip teachers to efficiently employ teaching strategies to advance learners' higher-order thinking skills as response to the demands of the changing character of 21st century learners:

Indicator 1.5.2	***Apply a range of teaching strategies to develop critical and creative thinking, as well as other higher-order thinking skills***
Indicator 1.5.3	***Develop and apply effective teaching strategies to promote critical and creative thinking, as well as other higher-order thinking skills***

This *Teacher's Resource* incorporates the principles of **S**tructure of the **O**bserved **L**earning **O**utcome (SOLO) taxonomy developed by Biggs and Collis (1982) to facilitate higher-order learning effectively among learners.

SOLO is developed to classify learning outcomes based on their complexities, allowing teachers to assess learners' learning outcomes in terms of quality. Moreover, it can be used as a framework to describe the levels of complexities in higher-order thinking skills. Learners with higher-order thinking skills demonstrate at least the relational level of complexity in SOLO.

What teachers can expect from this Resource Material

To assist you in responding to PPST Indicators 1.5.2 and 1.5.3, this *Teacher's Resource* contains **non-prescriptive** and **suggestive** SOLO-based items that you may use in the classroom.

This *Teacher's Resource*, hence, aims to support you in understanding HOTS and in reflecting these skills in your respective classroom practices. This, then, shall guide you in performing pedagogy and assessment practices in Mathematics that promote learners' critical thinking, creative thinking, and higher-order thinking skills.

In summary, this resource seeks to:

- ✓ **address the appropriate strands/indicators** in the:
  - **Philippine Professional Standards for Teachers** (PPST Strand 1.5 *Strategies for developing critical and creative thinking, as well as other higher-order thinking skills*);
  - **Philippine Professional Standards for School Heads** (PPSSH Strand 3.2 *Teaching Standards and Pedagogies*); and
  - **Philippine Professional Standards for Supervisors** (PPSS Strand 3.1 *Support for Instructional Leadership*);
- ✓ **advocate different learning approaches and modalities** through distance and blended (multi-modal) learning;
- ✓ **support the development and application of collaborative expertise** in teachers, master teachers and school heads to underpin their own development through the Learning Action Cells (LAC); online presentations, classroom applications and mentoring;
- ✓ **keep teachers abreast with various HR systems within DepEd** such as the demonstration of indicators in the RPMS through classroom observations;
- ✓ **assist in the teaching and learning process** and be able to respond seamlessly to the expectations set by international and national assessments; and
- ✓ **respond to professional development needs** identified in DepEd Memorandum 50, s. 2020, or the DepEd Professional Development Priorities for Teachers and School Leaders for School Year 2020-2023.

Introduction to the Basic SOLO Model

by Professor John Pegg

This Introduction to the basic SOLO Model, divided into four Parts, was written by Professor John Pegg of the SiMERR National Research Centre in Australia. The aim is to situate the reader within the early, and still highly relevant, research and thinking that has been undertaken on SOLO. This will enable teachers to develop a stronger base in assessing student responses. This is particularly relevant in the case of explaining lower-order and higher-order questioning and thinking.

Part 1 Background to SOLO

The SOLO Model (the SOLO Taxonomy) of John Biggs and Kevin Collis (Biggs & Collis 1982, 1991; Pegg 2003, 2020) is a cognitive (brain-based) developmental framework that offers a useful tool to explore the quality of a learner response in a specific context. The notion of 'quality' is not unfamiliar in Education discussions, its importance is seldom challenged. However, trying to tie down a meaning for quality and what it means operationally has shown not to be so easy.

At its basis, SOLO is interested in describing the nature of a learner response to a question or stimulus. This information offers insights into what a learner knows, understands and can do, as well as directions along which instruction may most profitably proceed.

When asked the 'quality' of a student's learning, a common response is to mention the number of facts or pieces of relevant information a person knows something about. This information might be further supported by citing scores on a recent examination, or the number of correct items a person has achieved in some test/quiz.

Such descriptions of 'quality', definitely offer a perspective on learning performance – a view that knowledge creation is about acquiring more and more pieces of information. In terms of operationalizing 'quality', this description can result in predictable and routine approaches to instruction involving drill and practice. However, this view can also limit a breadth of practices in teaching involving more demanding higher-order practices such as analysis, explanation, and synthesis, and that may lead to unfortunate long-term consequences for a learner.

Being told that a student obtained a score of 73% on a test tells us very little about the quality of the learning, except that the student probably knows more than someone who achieved 63% on the same test and not as much as someone who scored 83%. However, little can be interpreted if the comparison was with a person who achieved 70-72% OR 74-75%.

Data are clear that tests are often limited in their ability to discriminate meaningful student learning on scores within a few percentage points of one another. The impact of misguided interpretations of learning is even more dramatic when scores hover around the 50% mark. A mark which usually holds an unprecedented and undeserved importance by society.

Also, there are often issues interpreting student scores when students obtain the same score. Do similar scores on a test mean that students have the same questions correct or incorrect? Ideas of equivalence can be misleading. It is possible, for example, that one student earned their marks on the most straightforward questions across the test, while another respondent might be able to achieve correct responses on some quite difficult questions in certain areas and perform poorly in other areas.

It would seem quite likely that a student who is able to respond to some more difficult questions is likely to be able to advance more quickly with support, than a student who is only able to undertake the more basic questions correctly.

Further, and more importantly, numeric descriptions of quality do little to explain:

- what a learner knows or understands;
- in what directions a teacher, or the learner themselves, might move to improve or advance their learning; and, as importantly,
- how might this notion of 'quality' link to how the brain learns.

SOLO offers help in addressing these concerns. The focus of the SOLO categorization is on cognitive processes in addressing an issue or question rather than the end-products alone. SOLO offers a framework that enables explorations and descriptions of the quality of *'how well'* learning has progressed in different contexts. This provides a genuine balance to more typical approaches, mentioned earlier, that describe 'how much' is known.

The application of SOLO to the analysis of learner responses enables insights into learner cognitive development as well as understandings of possible cognitive blockages associated with the pattern of ideas that are impacting on leaner growth. As such, SOLO offers teachers insights into learner thinking and subsequent teaching actions.

Part 2 Overview of SOLO

Over the past 40 plus years, since the late 1970s, SOLO has built a substantial evidence base involving many thousands of research studies resulting in many hundreds of published articles. Now, SOLO has an extensive and growing universal following.

SOLO has emerged out as a consequence of describing learning through the eyes of a learner involving two separate but related activities. This involves:

- the acquisition or development of relevant ideas, facts, skills, concepts, processes and strategies; and
- the use of this acquired information in some form such as to solve problems, apply understanding, or explain or interpret meaning.

This reflects the two main ideas in Part 1 above concerning describing quality as 'how much' and 'how well'.

In terms of this current publication, this dual approach to thinking about 'quality' linked to SOLO, offers a realistic and practical description of what lower-order and higher-order thinking looks like as demonstrated in a learner's response.

In particular, SOLO enables teachers to distinguish between skills, knowledge and content that may be considered as lower-order functioning (or the result of surface learning) and those described as higher-order functioning (or the result of deep learning). SOLO supports teachers with ways to identify the practical meaning of lower-order and higher-order quality, and ways to identify examples in different contexts.

Such practical advice on applying decision skills, to distinguish lower- and higher-order functioning is achieved by describing an operationalized balance between:

(i) the degree of complexity of how responses are structured by the brain; and
(ii) relevant information associated with the content/context.

As lower-order skills and understandings are necessary pre-requisites for higher-order thinking, the ability of teachers to efficiently and effectively separate lower- and higher-order categorizations is a critically important skill. SOLO offers a structure upon which such decisions can be made.

This significant strength of the SOLO model lies in its links with neuroscience and how the brain learns, i.e., the cognitive (brain) processes. These brain-based ideas behind SOLO are linked to:

- information processing capacity, such as, working memory demands;
- the creation of neural pathways/networks through deliberate practice;
- the amount of information able to be retained by the learner in a particular domain; and,
- features specific to learning tasks or activities.

Overall, despite the obvious importance of the notion of 'quality' to education, descriptions of what is meant by quality have not received the attention, or use in practice, it deserves. SOLO offers an alternative to traditional assessment counts of 'how many', by placing SOLO center stage in learning and teaching. Teachers who learn to apply SOLO routinely in the classroom find that it is relevant and useful to understanding learning situations in all subject areas.

Further, when used correctly, SOLO can help teachers not only apply more *objective* and *systematic* assessment techniques, but it can help clarify developmental learning pathways to inform lesson and syllabus development, as well as strengthen formative-assessment approaches.

There are four main aspects to modern descriptions of SOLO. These are:

- SOLO levels
- the SOLO modes
- SOLO levels within modes
- SOLO cycles.

All four aspects are important for completeness, but initially, it is sufficient for the reader to become familiar with the meaning, use and application of the concepts around **SOLO levels (Section Part 3 below).** This feature is the one most prominent in early general discussions. Also, when information about SOLO is provided, say on the Web, the information provided on **SOLO levels** is usually the sole focus.

Part 3 Introduction to *SOLO Levels: Language and Meaning*

Biggs and Collis (1982) believe the way the brain structures learnt material, 'structural organization' of knowledge, is the difference between well learned from poorly learned material. It is this structural aspect of knowledge in the brain that underpins descriptions of quality. They (Biggs and Collis):

> believe that there are 'natural' stages in the growth of learning any complex material or skill… in certain important aspects these stages are similar to, but not identical with, the developmental stages in thinking described by Piaget and his co-workers. (Biggs & Collis, 1982, p. 15)

SOLO Levels

SOLO Levels are the most well-known aspect of the SOLO model. The SOLO levels describe the increasing sophistication (the increasing quality) of responses in handling certain tasks/questions relevant to a particular activity or domain. The levels are given specific names that every teacher needs to acquire and use accurately and consistently.

In the 1982 version of SOLO there are five levels of response. They represent a developmental continuum beginning from a level that describes an irrelevant or incorrect understanding, through a series of three levels describing how the brain structures understanding to an acceptable degree, to a fifth and final level where a response extends beyond what might typically be expected as an acceptable response.

Unistructural, Multistructural and Relational levels

In what follows the middle three levels are described first, and the first and last of the five levels are considered second. The three middle levels have the names unistructural, multistuctural and relational. The names are almost self-explanatory. Their level descriptions are:

A *unistructural* (U) response is one where the focus is on a single attribute. It might involve writing a single sentence with one main idea, or undertaking one algorithm, or providing one reason or suggestion, or identifying one relevant piece of information from the stimulus, etc. The key to this level is in the name. The prefix 'uni' stands for 'oneness'. So, the 'structure' of the response is a single aspect that is relevant to the question or activity.

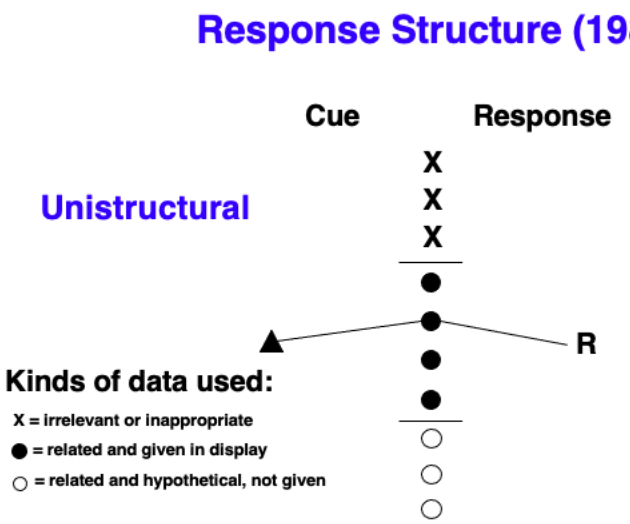


Figure 1: Unistructural level

A *multistructural* (M) response is one that includes several relevant independent pieces of information from the stimulus or comprises a number (i.e., more than 1) usually sequential actions, explanations, algorithms, etc. The key to this level is in the name. The prefix 'multi' stands for 'many'. So, the structure of the response contains more than one aspect that is relevant to the question or activity. Further the different aspects are seen to be independent of one another. There is no integration of pieces of information or seeing inter-related aspects.

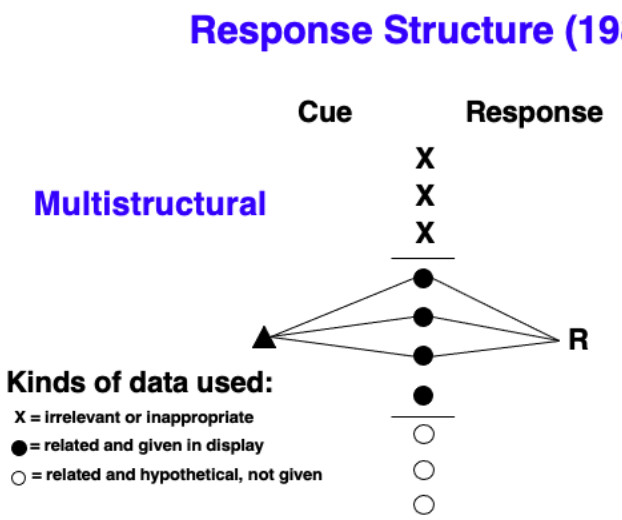


Figure 2: the Multistructural level

A *relational* (R) response is one that integrates all relevant pieces of information or data from the stimulus. These aspects in the stimulus are now linked to one another resulting in an overall coherence, a pattern, to the data presented and any approach to be undertaken. There is no inconsistency within the known system.

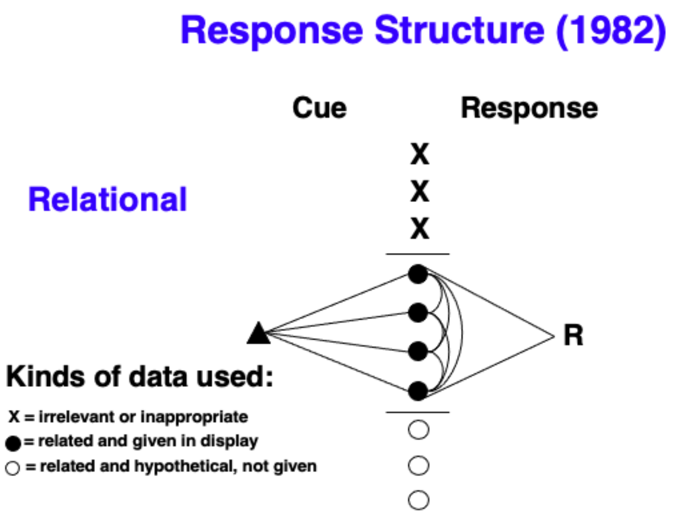
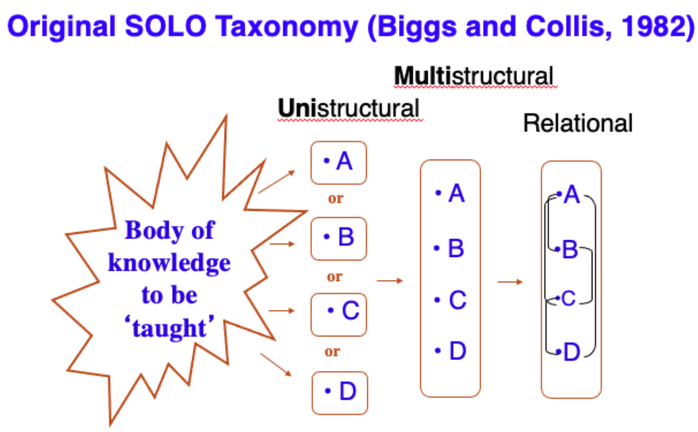


Figure 3: The Relational level

These three levels are often referred to as a SOLO unistructural–multistructural–relational cycle or a SOLO UMR cycle.



The three levels described above offer descriptions of increasing complex structures of thinking by the brain in which higher levels are directly built upon preceding levels, i.e., the mutlistructural response contains the unistructural response, a relational response identifies the relationships among the separate elements of the multistructural response. Taken together, the three levels represent a SOLO UMR cycle.

Prestructural and Extended Abstract levels

Two other SOLO levels can be found in the literature. They are most relevant to and used when people talk about the SOLO Taxonomy based around the 1982 book. The names of the two levels are prestructural and extended abstract.

The prestructural level, as the word indicates, occurs 'pre' or 'before' the structure starts and so it is used to code responses that fail to address a relevant feature. Such a response is described as:

A prestructural level (Pre) of response is one that does not focus on the relevant question or activity. Usually, the answer is quickly given with little thought. The answer is likely to be irrelevant or simply repeat information already provided in the question or activity.

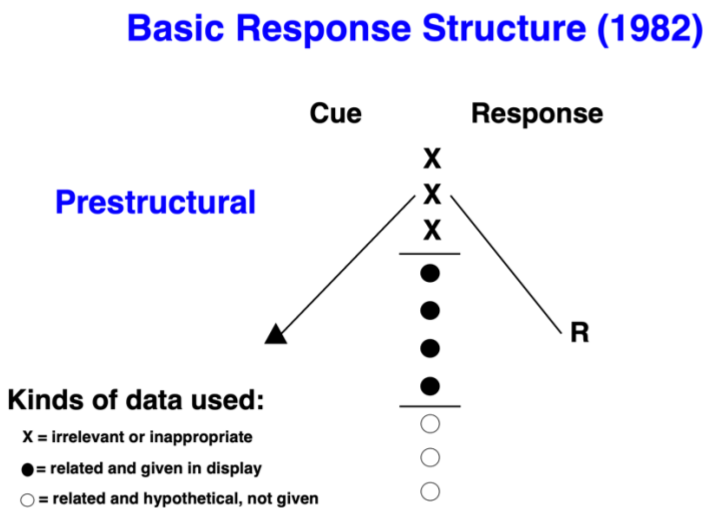


Figure 4: The Prestructural level

The extended abstract level, as the word meaning indicates, occurs after a relational response. So, it is used to identify a response that goes beyond what might typically be expected. In such cases the answer would have a deeper perhaps more abstract feel, hence, the name.

An extended abstract (EA) response is one that goes beyond what was expected at the relational level. In school situations it can involve deduction, ability to close on situations not experienced. Answers can be held open or qualified to allow for logical alternatives.

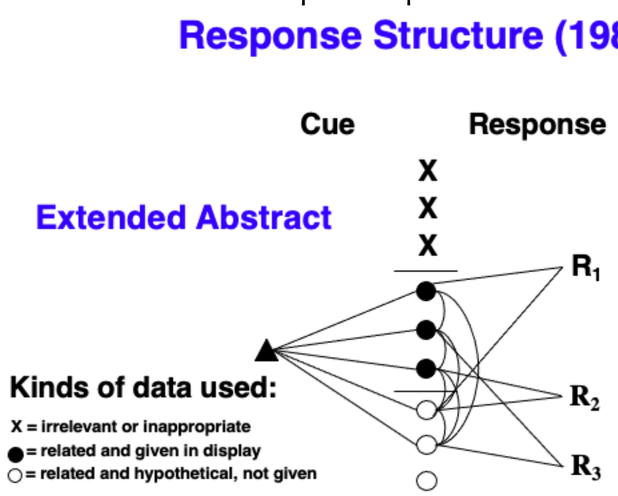


Figure 5: The Extended Abstract level

The two outer levels, one below and one above the middle three levels, respectively, are named prestructural and extended abstract.

Finally, a useful question is: What are the variables that determine or underpin the level of response given by a learner? There are five.

The Degree of Abstractness: The first level (pre-structural) is personal to the learner and not the topic. The next three levels (unistructural, multistructural and relational) are relevant to the area of focus and share similar characteristics. The last level (extended abstract) is more general and extends beyond the previous levels in an appropriate abstract way.

Number of Organizing Dimensions: The first level does not have an organizing dimension relevant to the activity or question. For the next three levels the organization is based on one dimension, several independent dimensions, and an integration of the independent dimension. The final level moves beyond the previous by adding an overarching general framework encompassing the earlier work.

Consistency: The first level is the most inconsistent. This encompasses the information provided and the response provided. The next two levels provide growing consistency as more elements are used in determining a response. The relational level response is consistent within the internal or provided context of the learner. The extended abstract level response not only is able to work within the internal context but can also consider external principles or other contexts providing a much deeper and often more nuanced response.

Openness of Conclusion: The list of levels demonstrates a graduation of thinking from 'closed' to 'open'. 'Closed' is where learners respond very quickly to an activity or stimulus, sometimes without even understanding the question. As a learner offers increasing levels of response, there is more time and consideration provided, i.e., the response becomes more 'open' so as to enable more room for considered interpretations. This 'openness' is maximized for extended abstract responses.

Sequence of Levels: The levels are developmental with an earlier level being a building block for the next level. A unistructural response is within the related multistructural response. A relational response integrates the elements of the earlier multistructural response. An extended abstract response has within it the relational response but extends it through embedding the response within a broader external environment or through incorporating broader principles or theoretical positions.

Part 4 The SOLO Model and Instruction

The strength of the SOLO model is the linking of the hierarchical nature of cognitive development through the modes (not mentioned in this Summary) and the cyclical nature of learning through the levels.

In terms of SOLO levels, each level provides building blocks for the next higher level. SOLO also provides teachers with a common and shared language that enables them to describe in a meaningful way their observations of student performance. This is particularly important when teachers try to articulate differences between lower-order and higher-order skills and understandings.

Emerging from careful research work of SOLO is the observation that while the lower levels in the SOLO model can be taught in the traditional sense. The shift to developing learner higher-order skills and for them to be able to respond to questions with higher-order responses requires a quality in the thinking of the learner that cannot be guaranteed by explicit teaching alone.

There appears to be certain teaching approaches and strategies that might be better applied when students are identified as responding at one SOLO level than when at another. Knowledge of this pattern can better help teachers develop a rationale for their actions and help inform the nature of their instruction to targeted groups.

Part 5 Final Comment

Overall, it has been clear that for the great majority of teachers, assessment of subjects taught in school are dominated by a focus on content (in the form of facts) and skills (associated with computational techniques), and the ability of learners to reproduce these on demand. This narrow focus can have a sterile effect upon innovations and developments in the mathematics curriculum and even on what it means for a person to think mathematically or scientifically.

The issue here for teachers is about

(i) interpreting the quality of the learning in terms of 'how well' material is understood (Biggs & Collis, 1982; 1991); and
(ii) selecting the most appropriate strategies, procedures or teaching activities for their students at their SOLO response level.

Higher-order goals of learning, such as judgement formation, solving relevant problems, and on developing understanding, must encompass not only the content, but also the interrelationships between various processes and procedures.

Nevertheless, these more demanding skills and developments must be built on fundamental lower-order knowledge, skills and understandings. Quality education, instruction and learner outcomes, must embrace the full range of abilities as described and categorized through the SOLO model in the topics identified.

Learning Approaches

This *Teacher's Resource* deals with varied stems, questions, or activities that you may integrate into your lesson delivery using different approaches. These approaches may include (1) Mathematical Modelling Approaches, (2) Critical Thinking and Problem-Solving based Approaches and, (3) Assessing Mathematical Literacy through Mathematical Reasoning & Computational Thinking. These strategies have been found to enable higher-order thinking among learners. You may also want to tap other related time-tested pedagogical approaches in teaching Mathematics.

Moreover, there are some procedures that you may consider. We categorize these procedures as Instruct-Situate-Question-Build strategies. This is a Problem-Based Learning Approach designed for the flow of SOLO-based questions to aid teachers. It starts with Instruct on what the teacher will expect on the Situate-Question-Build steps/components of the procedure. The Situate contains the context, which makes it PISA quality question/s; while the Question contains unistructural and/or multistructural level of questions. Lastly, the Build part contains the relational level questions, which was built further from unistructural and multistructural levels.

1. *Mathematical Modelling Approach*

Mathematical Modeling is an attempt to study and understand real-life problems using mathematics. It has been an integral part of society that plays a vital role in business, engineering, and many other fields (Vos, 2013). With its importance, it just makes sense that it is now integrated into schools. *Math Modelling* is the process of choosing and using appropriate mathematical ideas and concepts to analyze and solve problems (NCTM, 2016). Wessels (2014) defines *Mathematical Modelling* as the practice of making sense of the world through a mathematical perspective.

According to Boaler (2001), *mathematical modelling theory* focuses on individuals and suggests that knowledge is created as a result of a series of interactions between people and the world. This situation requires examination of learners' situations with different practices. For this reason, it becomes important to present learners with situations in which they use the knowledge and have application opportunities.

2. *Critical Thinking and Problem-Solving based Approaches*

The twin goals of K to 12 mathematics teaching education are critical thinking and problem-solving. Critical thinking, according to Scriven and Paul (1987), is the intellectually disciplined process of actively and skillfully conceptualizing, applying, analyzing, synthesizing, and/or evaluating information gathered from, or generated by, observation, experience, reflection, reasoning, or communication, as a guide to belief and action. On the other hand, according to Polya, mathematical problem solving is finding a way around a difficulty, around an obstacle, and finding a solution to a problem that is unknown.

3. Assessing Mathematical Literacy through Mathematical Reasoning & Computational Thinking

PISA 2021 offers a framework for assessing mathematical literacy in the 21st century, which includes mathematical reasoning and some aspects of computational thinking. These problems in this guide will be solved using mathematical reasoning and computational thinking with the aid of SOLO. The assessment is aimed at presenting problems assessing Mathematics Literacy using mathematical reasoning and computational thinking. More specifically, it aims

1. To be able to present problems that involve Mathematical Reasoning.
2. To be able to present problems that involve Computational Thinking.
3. To be able to present these problems using SOLO Learning Approach.

Enacting Mathematical Approaches

According to Tatto (2008), there are three hypothesized subdomains for the framework for mathematics pedagogical content knowledge: mathematics curricular knowledge, knowledge of planning mathematics, and knowledge of enacting mathematics (p.4). Mathematical enactment focuses on analysis and evaluation of learners' mathematical solutions or arguments as well as contents of learners' questions. It also includes diagnosis and analysis of typical learners' responses, including misconceptions as well as explaining or representing mathematical concepts or procedures. It is also a venue of generating fruitful questions and responding to it by providing appropriate feedback.

In enacting mathematics for teaching and learning, activities may include but are not limited to (1) analyzing or evaluating learners' mathematical solutions or arguments; (2) analyzing the content of learners' questions; (3) diagnosing typical learners' responses, including misconceptions; (4) explaining or representing mathematical concepts or procedures; (5) generating fruitful questions; (6) responding to unexpected mathematical issues; and (7) providing appropriate feedback.

In Mathematics Enacting, teachers may ask learners to find where mistakes have been made among the given answers/solutions and explain why they were mistakes. Have learners provide the correct answer to the question. Learners may also allocate marks for the wrong answer/s identified for added fun in the activity.

In the following illustration, the SOLO level of questioning is multistructural. Here, teachers may think of one lower-order activity at the multistructural level in which different methods, approaches, or techniques can be used to find or provide an answer.

Addressing Challenges

Integration of the SOLO model and various teaching approaches poses challenges to teachers. Vital attention for teaching text structures can be associated with high stakes assessments at the regional, national, and international arena in secondary/high school level measures (e.g., National Achievement tests, PISA).

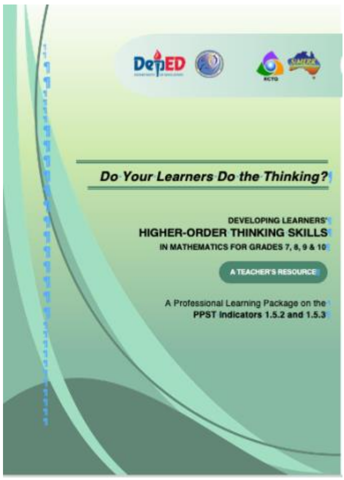
Generally, the types of questions on these tests are categorized to measure higher-order thinking skills. These questions typically focus on causes for the problem, effects of the solutions, and even comparing alternative solutions.

Another challenge is the relation of metacognition to content. It is important that learners learn the content before they can apply metacognition (Lin, Schwartz, & Hatano, 2005). This is why teachers need to ensure that content is learned by learners.

For face-to-face classes, the teacher can ask a series of questions ranging from *unistructural* to *multistructural*, *relational* and *extended abstract* response questions. The same can be done in an online modality. However, this becomes a challenge for other modes where no interaction between the teacher and the learner is possible such as with modular instruction.

For big class sizes, maximum learner participation can be tapped by throwing structured questions to the group. SOLO-based questions may be asked, and volunteer learners will be called to recite. Class discussion will help and hopefully yield desired results.

If learning is your passion, this Professional Learning Package ***is for you.***



Grade 7

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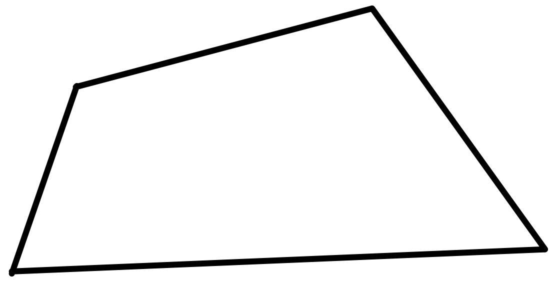
Item No. 001	Subject	Mathematics – Geometry
	Grade Level	7
	Topic	Angles of Polygons (part 1)
	PISA Competency	Relates to the understanding of spatial and geometric phenomena and relationships.
	K to 12 Curriculum Competency	M7GE-IIIf-1: The learner derives inductively the relationship of exterior and interior angles of a convex polygon. **Content Standards and Performance Standard:** • *The learner* demonstrates understanding of key concepts of geometry of shapes and sizes, and geometric relationships. • *The learner* is able to create models of plane figures and formulate and solve accurately authentic problems involving sides and angles of a polygon.
	Higher Order Thinking Strategy adopted	✓ Explicitly using SOLO (from U → M → R) ✓ Alternative question taking a different approach, method, or technique ☐ Presenting information in a different form ☐ Reversing the way that a solution is traditionally achieved ☐ Providing an erroneous answer and asking students to correct mistakes ☐ Compare and contrast two similar features ☐ Other

ACTIVITY 1 Grade 7 Mathematics: Geometry (Polygons, part 1)

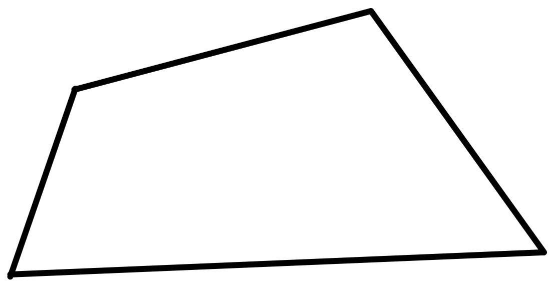
Question 1 Using you prior knowledge that the interior angle sum of a triangle measures 180°:

Question 1 a1. Describe how you would you be able to find the sum of the interior angles of a quadrilateral? Show your working.



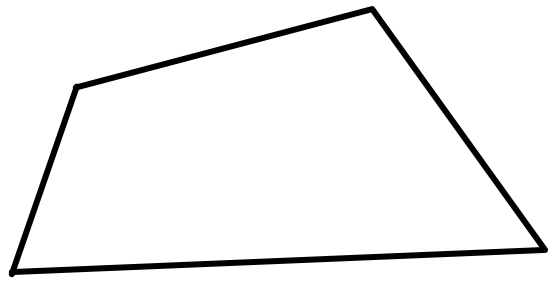
Your Answer:

Question 1 b1. Draw three triangles that do not overlap but still fill the space inside the given quadrilateral.



Your Answer:

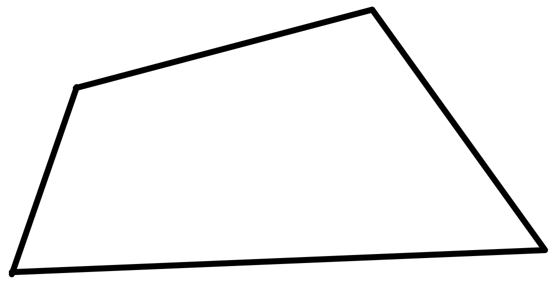
Question 1 b2. Describe how you would be able to find the sum of the interior angles of a quadrilateral using the quadrilateral that has been divided into ***three*** non-overlapping triangles. Show/explain you working.



Your Answer:

Question 1 c1.

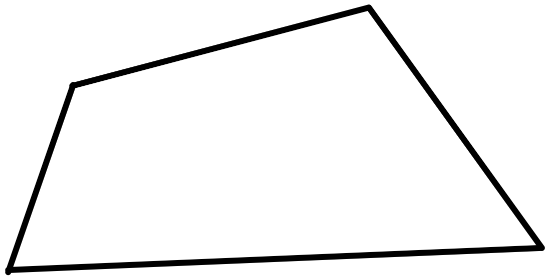
Using triangles in the interior of a quadrilateral from any point in its interior: What would be the solution in getting the sum of the interior angles of the quadrilateral?



Your Answer:

Question 1 c2.

Consider the case (if not done so already) where the triangles within the quadrilateral were overlapping. Students would be required to use three or more triangles to determine the angle sum of a quadrilateral.



Your Answer:

Item No. 002	Subject	Mathematics – Geometry
	Grade Level	7
	Topic	Angles of Polygons (part 2)
	PISA Competency	Relates to the understanding of spatial and geometric phenomena and relationships.
	K to 12 Curriculum Competency	M7GE-IIIf-1: The learner derives inductively the relationship of exterior and interior angles of a convex polygon. **Content Standards and Performance Standard:** • The learner demonstrates understanding of key concepts of geometry of shapes and sizes, and geometric relationships. • The learner is able to create models of plane figures and formulate and solve accurately authentic problems involving sides and angles of a polygon.
	Higher Order Thinking Strategy adopted	☐ Explicitly using SOLO (from U → M → R) ✓ Alternative question/item taking a different approach, method, or technique ☐ Presenting information in a different form ☐ Reversing the way that a solution is traditionally achieved ☐ Providing an erroneous answer and asking students to correct mistakes ✓ Compare and contrast two similar features ☐ Other:

ACTIVITY 2 Grade 7 Mathematics: Geometry (Polygons, part 2)

Question 2 a1. Complete the following table to find the sum of interior angles of pentagon, hexagon, and octagon. They may use as many solutions as possible.

Polygon	Number of Sides	Smallest Number of non-overlapping triangles	Sum of Interior Angles
Quadrilateral			
Pentagon			
Hexagon			
Heptagon			
Octagon			

Question 2 b1. Using the information in the table or otherwise find the sum of the interior angles of a polygon with

(i) 12 sides.
(ii) 30 sides.

Your Answer:
(i)

(ii)

Question 2 b2. Using the information in the table of otherwise, find the number of sides of a polygon if the interior angle sum is:

(i) 2,700°
(ii) 7,560°

Your Answer:
(i)

(ii)

Question 2 b3. Write a sentence expressing the relationship (rule) that links the number of sides of a polygon to the angle sum of a polygon.

Your Answer:

Question 2 c1. If the number of sides of a polygon is '*y*' and interior angle sum is '*x*', express this relationship as an equation.

Your Answer:

Question 2 c2. Check that the formula developed is true for the cases listed to the answers in **Question 2 a1.**

Your Answer:

Question 2 c3. What would be the sum of interior angles of a polygon with

(i) 50 sides?

(ii) 300 sides?

Your Answer:

(i)

(ii)

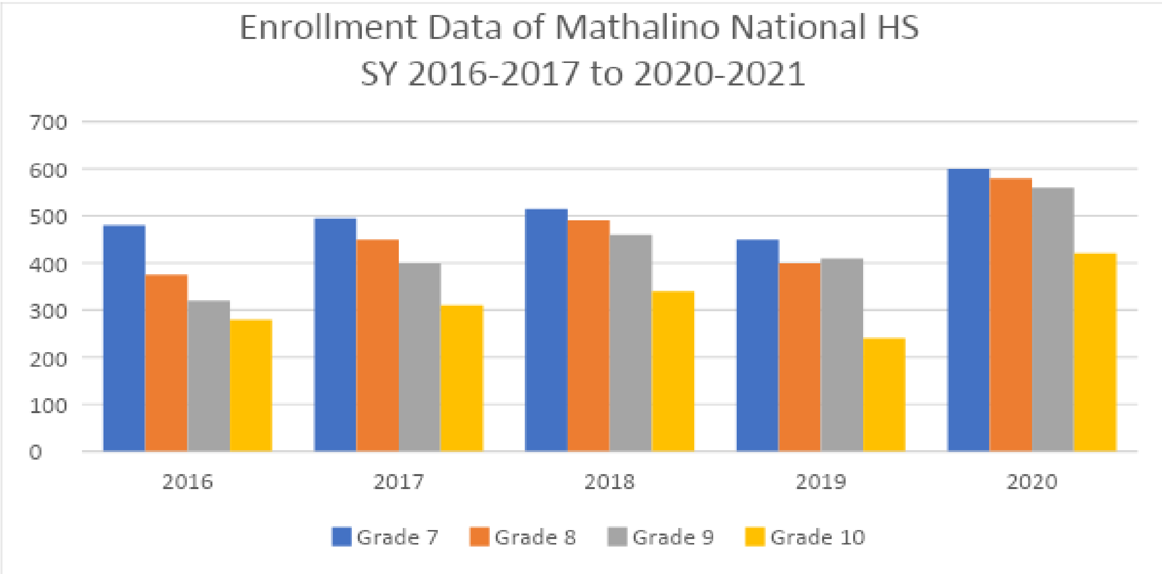
Question 2 c4. Which solution strategy would be the ‘best’ one to use for such large numbers of sides? Why? Give reasons for your answer.

Your Answer:

Item No. 003	Subject	Mathematics – Statistics and Probability
	Grade Level	7
	Topic	Bar Graphs
	PISA Competency	Understanding variation as the heart of statistics.
	K to 12 Curriculum Competency	M7SP-IVd-e-1: *The learner* uses appropriate graphs to represent organized data: pie chart, bar graph, line graph, and histogram. **Content Standards and Performance Standard:** • *The learner* demonstrates understanding of key concepts, uses and importance of Statistics, data collection/gathering and the different forms of data representation, measures of central tendency, measures of variability, and probability. • *The learner* is able to collect and organize data systematically and compute accurately measures of central tendency and variability and apply these appropriately in data analysis and interpretation in different fields.
	Higher Order Thinking Strategy adopted	✓ Explicitly using SOLO (from U → M → R) ☐ Alternative question/item taking a different approach, method, or technique ☐ Presenting information in a different form ☐ Reversing the way that a solution is traditionally achieved ☐ Providing an erroneous answer and asking students to correct mistakes ✓ Compare and contrast two similar features ☐ Other:

ACTIVITY 3 Grade 7 Mathematics: Bar Graphs

Question 3. Mr. Marunong is the school head at Mathalino National High School, one of the leading math schools in the provinces. He wants to determine the number of students at the school for the past five years to develop a program that will benefit the students, teachers, and school. One of the teachers gave him a graph that shows the information for the school years 2016-2020.



Question 3 a1. What year does the school have the highest number of enrollees

Your Answer:

Question 3 a2. Which of the following statements is/are true about the bar graph?

a. The number of students is increasing over a period of 5 years
b. There was a decrease in students during 2019
c. The year 2020 has the highest number of students
d. Grade 10 was the least number of students among grade levels.

Your Answer:

Question 3 a3. What are the main messages/story that the given bar graph tells us on the student numbers?

Your Answer:

Item No. 004	Subject	Mathematics
	Grade Level	7
	Topic	Numbers and Number Sense
	PISA Competency	Employing Mathematical Concepts, Facts, Procedures, and Reasoning 1. *devising and implementing strategies for finding mathematical solutions* 2. *manipulating numbers, graphical and statistical data and information, algebraic expressions and equations, and geometric representations*. Represent real-life situations and solve problems involving real number
	K to 12 Curriculum Competency	M7NS-Ic-d-1: *The learner* performs fundamental operations on integers. **Content Standards and Performance Standard:** • *The leaner* demonstrates understanding of key concepts of sets and the real number system. • *The learner* is able to formulate challenging situations involving sets and real numbers and solve these in a variety of strategies.
	Higher Order Thinking Strategy adopted	✓ Explicitly using SOLO (from U → M → R) ☐ Alternative question/item taking a different approach, method, or technique ☐ Presenting information in a different form ☐ Reversing the way that a solution is traditionally achieved ✓ Providing an erroneous answer and asking students to correct mistakes ☐ Compare and contrast two similar features ☐ Other:

ACTIVITY 4 Grade 7 Mathematics: Number and Number Sense

Question 4. June was asked to simplify the given expression involving operations on integers [(+81) ÷ (-9) + (-4)] – [2(-4) + (+6)]. His solution was given below.

Student (June's) Attempt:

1 [(+81) ÷ (-9) + (-4)] – [(2) (-4) + (+6)]
2 [(-9) + (-4)] – [(2) (-4) + (+6)]
3 [(-9) + (-4)] – [(-8) + (+6)]
4 [-9 -4] – [-8 + 6]
5 [-13] – [2]
6 -13 – 2
7 = -15

Question 4 a1. Is the solution correct? Justify your answer.

Your Answer:

Question 4 a2. If you were marking this question out of five marks, how many marks would you give the student. Provide a number between 0 and 5 marks and describe why you allocated this mark.

Your Answer:

Item No. 005	Subject	Mathematics
	Grade Level	7
	Topic	Weight
	PISA Competency	Recognizing functional relationships between quantities.
	K to 12 Curriculum Competency	M7AL-IIj-2: *The learner* solves problems involving equations and inequalities in one variable. **Content Standards and Performance Standard:** • *The learner demonstrates understanding of key concepts of algebraic expressions, the properties of real numbers as applied in linear equations, and inequalities in one variable.* • *The learner is able to model situations using oral, written, graphical, and algebraic methods in solving problems involving algebraic expressions, linear equations, and inequalities in one variable.*
	Higher Order Thinking Strategy adopted	✓ Explicitly using SOLO (from U → M → R) ☐ Alternative question/item taking a different approach, method, or technique ☐ Presenting information in a different form ✓ Reversing the way that a solution is traditionally achieved ☐ Providing an erroneous answer and asking students to correct mistakes ☐ Compare and contrast two similar features ☐ Other:

ACTIVITY 5 Grade 7 Mathematics: Weight

Question 5

Sarah joined a community outreach last Sunday. She was in-charge of packing fruits particularly ponkans before putting them in crates. Each crate weighs 3 kilograms when empty, an individual ponkan averages 0.2 kilograms. When the crate is full it weighs 12 kilograms.

Question 5 a1.

What is the total weight of ponkan placed in the crate?

Your Answer:

Question 5 a2.

How many ponkans can Sarah place in a full crate?

Your Answer:

Question 5 a3.

At another farm, the packers found that they could fit 50 ponkans in a crate with the same weight. How much did their ponkans weigh?

Your Answer:

<table>
<tr><th rowspan="7">Item No. 006</th><th>Subject</th><th>Mathematics</th></tr>
<tr><td>Grade Level</td><td>7</td></tr>
<tr><td>Topic</td><td>Measures of Central Tendency (part 1)</td></tr>
<tr><td>PISA Competency</td><td>Employing Mathematical Concepts, Facts, Procedures and Reasoning</td></tr>
<tr><td>K to 12 Curriculum Competency</td><td>M7SP-IVf-g-1: The learner calculates the measures of central tendency of grouped and ungrouped data.

Content Standards and Performance Standard:
• The learner demonstrates understanding of key concepts, uses and importance of Statistics, data collection/gathering and the different forms of data representation, measures of central tendency, measures of variability, and probability.
• The learner is able to collect and organize data systematically and compute accurately measures of central tendency and variability and apply these appropriately in data analysis and interpretation in different fields.</td></tr>
<tr><td>Higher Order Thinking Strategy adopted</td><td>☐ Uses an approach within a question of answering more difficult SOLO type-questions from Unistructural to Relational.
☐ Explicitly using SOLO (from U → M → R)
☐ Alternative question/item taking a different approach, method, or technique
☐ Presenting information in a different form
☐ Reversing the way that a solution is traditionally achieved
☐ Providing an erroneous answer and asking students to correct mistakes
☐ Compare and contrast two similar features
☐ Other:</td></tr>
</table>

ACTIVITY 6 Grade 7 Mathematics: Measures of Central Tendencies part 1)

Question 6.
Ruth is a Grade 7 student from Maliga High School. Her test scores out of a possible 100 in Mathematics were as follows:

78 89 87 79 80 78 76 88

Question 6 a1. Determine the range of test scores of Ruth in Mathematics

Your Answer:

Question 6 a2. Determine the average test score of Ruth in Mathematics.

Your Answer:

Question 6 a3.

There will be an additional test in Mathematics in one week. What should be the minimum grade Ruth needs to receive in order to increase her average test score of 1 point?

Your Answer:

Question 6 a4.

Is it still possible for Ruth to increase her average Mathematics test score to 2 points?

Your Answer:

<table>
<tr><th rowspan="7">Item No. 007</th><th>Subject</th><th>Mathematics</th></tr>
<tr><td>Grade Level</td><td>Grade 7</td></tr>
<tr><td>Topic</td><td>Measures of Central Tendency (part 2)</td></tr>
<tr><td>PISA Competency</td><td>Employing Mathematical Concepts, Facts, Procedures and Reasoning</td></tr>
<tr><td>K to 12 Curriculum Competency</td><td>M7SP-IVf-g-1: The learner calculates the measures of central tendency of grouped and ungrouped data.

Content Standards and Performance Standard:
• The learner demonstrates understanding of key concepts, uses and importance of Statistics, data collection/gathering and the different forms of data representation, measures of central tendency, measures of variability, and probability.
• The learner is able to collect and organize data systematically and compute accurately measures of central tendency and variability and apply these appropriately in data analysis and interpretation in different fields.</td></tr>
<tr><td>Higher Order Thinking Strategy adopted</td><td>☐ Explicitly using SOLO (from U → M → R)
☐ Alternative question/item taking a different approach, method, or technique
☐ Presenting information in a different form
☐ Reversing the way that a solution is traditionally achieved
☐ Providing an erroneous answer and asking students to correct mistakes
☐ Compare and contrast two similar features
☐ Other:</td></tr>
</table>

ACTIVITY 7 Grade 7 Mathematics: Measures of Central Tendancy, part 2)

Question 7. Richard sells fresh mangoes from his farm with an average weight of 152 grams. A sample of 18 mangoes yielded the following weights in grams:

148	149	152	152	151	152
154	153	150	149	148	145
152	156	148	139	151	149

Question 7 a1. What is the most common weight of the 18 mangoes selected for the customer?

Your Answer:

Question 7 a2. If the mangoes were laid out in order of their weights: What is the medium weight of the selected sample?

Your Answer:

Question 7 a3. Determine the difference between the advertised weight and the observed weight based on the sample

Your Answer:

Question 7 a4. Is it possible to attain an observed mean weight of 152 grams if another mango is added to the sample?

Your Answer:

Question 7 a5. Which measure of central tendency should be used to describe the data? Provide as much detail as possible in your answer.

Your Answer:

Item No. 008	Subject	Mathematics
	Grade Level	Grade 7
	Topic	Language of Sets
	PISA Competency	Recognizing functional relationships between quantities.
	K to 12 Curriculum Competency	M7NS-Ib-2: *The learner* solves problems involving sets. **Content Standards and Performance Standard:** • *The learner demonstrates understanding of key concepts of sets and the real number system.* • *The learner is able to formulate challenging situations involving sets and real numbers and solve these in a variety of strategies.*
	Higher Order Thinking Strategy adopted	☐ Explicitly using SOLO (from U → M → R) ✓ Alternative question/item taking a different approach, method, or technique ☐ Presenting information in a different form ☐ Reversing the way that a solution is traditionally achieved ☐ Providing an erroneous answer and asking students to correct mistake ☐ Compare and contrast two similar features ✓ Other: Develop a concept map

ACTIVITY 8 Grade 7 Language of Sets

Question 8.
Provide a list of words that you use when working involved with the concept of 'sets'.

Question 8 a1. Individual students should write down their own list.

Your Answer:

Question 8 a2. Groups of students should join together and (i) share individual ideas with an explanation and (ii) create a combined list to share with the class.

Your Answer:

Question 8 a3. Working on the board the class will combine all lists so that there is a master list.

Your Answer:

Question 8 a4. Develop a concept map.

Using the agreed class list of words on sets, organise them using a concept map. Student return to their group to develop their concept map. Students can use chart, arrows, or any images to show their ideas until the Group will form a concept map in Set and be ready to share this with the class.

Your Answer:

Grade 8

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	Subject	Mathematics
Item No. 001	Grade Level	Grade 8
	Topic	Triangle side lengths
	PISA Competency	Recognizing functional relationships between quantities
	K to 12 Curriculum Competency	M8GE-IVb-1: *The learner* applies theorems on triangle inequalities. **Content Standards and Performance Standard:** • *The learner demonstrates understanding of key concepts of inequalities in a triangle, and parallel and perpendicular lines.* • *The learner is able to communicate mathematical thinking with coherence and clarity in formulating, investigating, analyzing, and solving real-life problems involving triangle inequalities, and parallelism and perpendicularity of lines using appropriate and accurate representations.*
	Higher Order Thinking Strategy adopted	☐ Explicitly using SOLO (from U → M → R) ✓ Alternative question/item taking a different approach, method, or technique ☐ Presenting information in a different form ✓ Reversing the way that a solution is traditionally achieved ☐ Providing an erroneous answer and asking students to correct mistakes ☐ Compare and contrast two similar features ☐ Other:

ACTIVITY 1 Grade 8 Mathematics: Triangle side lengths

Question 1. Bernard is constructing a wooden triangular frame to be used as a part of a mold. He already has pre-cut wooden blocks which are 5 cms and 6 cms long to be used as 2 sides of the triangular frame. If he decides that the length of the third side is also a whole number solve the following questions.

Question 1 a1. By considering the Triangle Inequality Theorem, which states that the sum of the lengths of any 2 sides of a triangle must be greater than the third side; list down the possible length of the third side for Bernard to create the triangular frame.

Your Answer:

Question 1 a2. Using the answer to **Question 11 a1,** determine all the possible perimeters of the triangle.

Your Answer:

Question 1 b1. Suppose Bernard only had a 10-in wooden block to create the three sides of the triangular shaped mold, how should he cut the wooden block such that all measurements should be a whole number?

Note: The Triangle Inequality Theorem: This theorem states that the sum of the lengths of any 2 sides of a triangle must be greater than the third side.

Your Answer:

Item No. 002	Subject	Mathematics – Statistics and Geometry
	Grade Level	Grade 8
	Topic	Language of Probability
	PISA Competency	Representing a situation mathematically, using appropriate variables, symbols, diagrams, and standard models Choosing among an array of and employing the most effective computing tool to portray a mathematical relationship inherent in a contextualized problem
	K to 12 Curriculum Competency	M8SP-IVi-j-1: *The learner* solves problems involving probabilities of simple events. **Content Standards and Performance Standard:** • *The learner demonstrates understanding of key concepts of probability.* • *The learner is able to formulate and solve practical problems involving probability of simple events.*
	Higher Order Thinking Strategy adopted	☐ Explicitly using SOLO (from U → M → R) ✓ Alternative question/item taking a different approach, method, or technique ✓ Presenting information in a different form ☐ Reversing the way that a solution is traditionally achieved ☐ Providing an erroneous answer and asking students to correct mistakes ☐ Compare and contrast two similar features ✓ Other: Develop a concept map

ACTIVITY 2 Grade 8 Mathematics: Language of Probability

Question 2.

Provide a list of words that you use when working involved with the concept of 'probability'.

Question 2 a1. Individual students should write down their own list of words that come to mind when we mention the word 'probability'.

Your Answer:

Question 2 a2. Groups of students should join together and (i) share individual ideas with an explanation and (ii) create a combined list to share with the class.

Your Answer:

Question 2 a3. Working on the board the class will combine all lists so that there is a master list.

Your Answer:

Question 2 a4. Develop a concept map.
Using the agreed class list of words on probability, organize them using a concept map. Student return to their group to develop their concept map. Students can use chart, arrows, or any images to show their ideas until the Group will form a concept map in Probability and be ready to share this with the class.

Your Answer:

Grade 9

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<table>
<tr><th rowspan="7">Item No. 0 01</th><th>Subject</th><th>Mathematics – Geometry</th></tr>
<tr><td>Grade Level</td><td>Grade 9</td></tr>
<tr><td>Topic</td><td>Trigonometry in Oblique Triangle</td></tr>
<tr><td>PISA Competency</td><td>Appreciating the power of abstraction and symbolic representation.</td></tr>
<tr><td>K to 12 Curriculum Competency</td><td>M9GE-IVh -j-1: The learner solves problems involving oblique triangles.

Content Standards and Performance Standard:
• The learner demonstrates understanding of the basic concepts of trigonometry.
• The learner is able to apply the concepts of trigonometric ratios to formulate and solve real-life problems with precision and accuracy.</td></tr>
<tr><td>Higher Order Thinking Strategy adopted</td><td>✓ Explicitly using SOLO (from U → M)
✓ Alternative question/item taking a different approach, method, or technique
☐ Presenting information in a different form
☐ Reversing the way that a solution is traditionally achieved
☐ Providing an erroneous answer and asking students to correct mistakes
☐ Compare and contrast two similar features
✓ Other: Consolidating the meaning of a multistructural question</td></tr>
</table>

ACTIVITY 1 Grade 9 Mathematics: Geometry (Trigonometry in Oblique Triangle)

Question 1. Two friends Allan and Dan, each riding his own car, left Town A at the same time. Allan traveled along a straight road to Town B at an average speed of 60 kph, while Dan traveled along a straight road to Town C at an average speed of 90 kph and both reached their destinations after 45 minutes.

Question 1 a1. What is the total distance traveled by these two friends?

Your Answer:

Question 1 a2. If the road going to Town B forms a 120° angle with the road going to Town C, draw a simple representation of this information labelling key aspects.

Your Answer:

Question 1 a3. Determine how far is Town B from Town C if they are joined by a straight road?

Your Answer:

Question 1 a4. If Town C and Town B are connected by a straight road, how long will it take to travel at an average speed of 60 kph?

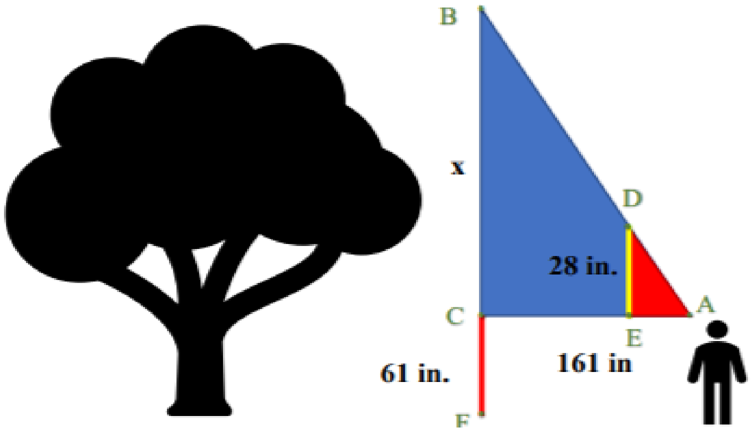
Your Answer:

Item No. 002	Subject	Mathematics – Geometry
	Grade Level	Grade 9
	Topic	Trigonometry, Right Triangles
	PISA Competency	Identifying the mathematical aspects of a problem situated in a real-world context and identifying the significant variables.
	K to 12 Curriculum Competency	M9GE-IIIj-1: *The learner* solves problems that involve triangle similarity and right triangles. **Content Standards and Performance Standard:** • *The learner demonstrates understanding of key concepts of parallelograms and triangle similarity.* • *The learner is able to investigate, analyze, and solve problems involving parallelograms and triangle similarity through appropriate and accurate representation.*
	Higher Order Thinking Strategy adopted	✓ Explicitly using SOLO (from U → M → R) ☐ Alternative question/item taking a different approach, method or technique ☐ Presenting information in a different form ✓ Reversing the way that a solution is traditionally achieved ☐ Providing an erroneous answer and asking students to correct mistakes ☐ Compare and contrast two similar features ☐ Other:

ACTIVITY 2 Grade 9 Mathematics: Geometry (Trigonometry, Right Triangles)

Question 2. Ivan is working on his Mathematics project. The project gives practical hands-on experience using similar triangles. Ivan chose JD Wile's stick method.

This method uses a stick that is the same length as his arm to make an isosceles triangle. Also, his arm is parallel to the ground, and the stick is perpendicular to his hand to form a right triangle as shown in the figure below with the indicated measures.



Question 2 a1. Based on the given diagram, prove that the two triangles are similar.

Your Answer:

Question 2 a2. Based on the given diagram, if Ivan's height to his shoulders is 61 inches, the length of the stick is 28 inches and the distance of Ivan from the tree is 161 inches, calculate the height of the tree in meters.

Your Answer:

Question 2 a3. Ivan decided to apply the same approach to another tree in his yard. His parents informed him that the height of this second tree was already known, and it was 121 inches. How far (in meters) does Ivan need to stand from the tree to verify this distance?

Your Answer:

<table>
<tr><td rowspan="6">Item No. 003</td><td>Subject</td><td>Mathematics – Patterns and Algebra</td></tr>
<tr><td>Grade Level</td><td>Grade 9</td></tr>
<tr><td>Topic</td><td>Quadratic Functions</td></tr>
<tr><td>PISA Competency</td><td>Using mathematical modelling as a lens onto the real world.</td></tr>
<tr><td>K to 12 Curriculum Competency</td><td>M9AL-Ii-j-2: The learner analyzes the effects of changing the values of a, h and k in the equation $y = a(x - h)^2 + k$ of a quadratic function on its graph.

Content Standards and Performance Standard:
• The learner demonstrates understanding of key concepts of quadratic equations, inequalities and functions, and rational algebraic equations.
• The learner is able to investigate thoroughly mathematical relationships in various situations, formulate real-life problems involving quadratic equations, inequalities and functions, and rational algebraic equations and solve them using a variety of strategies.</td></tr>
<tr><td>Higher Order Thinking Strategy adopted</td><td>✓ Explicitly using SOLO (from U → M → R)
✓ Alternative question/item taking a different approach, method, or technique
☐ Presenting information in a different form
☐ Reversing the way that a solution is traditionally achieved
☐ Providing an erroneous answer and asking students to correct mistakes
☐ Compare and contrast two similar features
☐ Other:</td></tr>
</table>

ACTIVITY 3 Grade 9 Mathematics: Quadratic Functions

Question 3 a1. Write $5x^2 - 30x + 12$ in the form $a(x - h)^2 + k$

Your Answer:

Question 3 a2. A rectangle has side lengths of $3y$ and $y+4$. The area of the rectangle is 36 cm^2. Show that $3(y + 2)^2 - 48 = 0$

Your Answer:

Question 3 a3. Find the perimeter of the rectangle.

Your Answer:

Item No. 004	Subject	Algebra and Patterns
	Grade Level	Grade 9
	Topic	Quadratic Equations
	PISA Competency	*Recognizing functional relationships between quantities*
	K to 12 Curriculum Competency	M9AL-Ia-b-1: *The learner* solves quadratic equations by (a) extracting square roots; (b) factoring; (c) completing the square; and (d) using the Quadratic formula. **Content Standards and Performance Standard:** • *The learner demonstrates understanding of key concepts of quadratic equations, inequalities and functions, and rational algebraic equations.* • *The learner is able to investigate thoroughly mathematical relationships in various situations, formulate real-life problems involving quadratic equations, inequalities and functions, and rational algebraic equations and solve them using a variety of strategies.*
	Higher Order Thinking Strategy adopted	✓ Explicitly using SOLO (from M → R) ☐ Alternative question/item taking a different approach, method, or technique ☐ Presenting information in a different form ☐ Reversing the way that a solution is traditionally achieved ✓ Providing an erroneous answer and asking students to correct mistakes ☐ Compare and contrast two similar features ☐ Other:

ACTIVITY 4 Grade 9 Mathematics: Quadratic Equations

Question 4. Jose was requested to solve for the roots of the quadratic equation by using the completing the square method. When his teacher checked his work, Jose learned that he was incorrect.

Jose's answer

STEP 1: Given: $3x^2 - 7x - 6 = 0$

$3x^2 - 7x = 6$

STEP 2: $x^2 - \frac{7}{3}x = 2$

STEP 3: $x^2 - \frac{7}{3}x + \frac{49}{36} = 2 + \frac{49}{36}$

STEP 4: $(x + \frac{7}{6})^2 = \frac{121}{36}$

STEP 5: $\sqrt{(x + \frac{7}{6})^2} = \pm\sqrt{\frac{121}{36}}$

$x + \frac{7}{6} = \pm\frac{11}{6}$

STEP 6: $x = -\frac{7}{6} \pm \frac{11}{6}$

$x = -\frac{7}{6} + \frac{11}{6}; x = -\frac{7}{6} - \frac{11}{6}$

Jose's final answer: $x = -3,\ x = \frac{2}{3}$

Question 4 a1. In which of the steps did Jose make his first mistake? Explain your answer and give the correct solution to the problem.

Your Answer:

Question 4 a2. Write out the correct solution to the problem.

Your Answer:

Question 4 a3. Determine what mark out of ten would have been **a fair reward** for Jose for the work he had undertaken? Given the reasons for your decision.

Your Answer:

Item No. 005	Subject	Mathematics – Patterns and Algebra
	Grade Level	Grade 9
	Topic	Radical Expressions
	PISA Competency	*Understanding quantity, number systems and their algebraic properties*
	K to 12 Curriculum Competency	M9AL-IIg-1: *The learner* simplifies radical expressions using the laws of radicals. **Content Standards and Performance Standard:** • *The learner demonstrates understanding of key concepts of variation and radicals.* • *The learner is able to formulate and solve accurately problems involving radicals.*
	Higher Order Thinking Strategy adopted	☐ Explicitly using SOLO (from U → M → R) ☐ Alternative question/item taking a different approach, method or technique ✓ Presenting information in a different form ☐ Reversing the way that a solution is traditionally achieved ✓ Providing an erroneous answer and asking students to correct mistakes ☐ Compare and contrast two similar features ☐ Other:

ACTIVITY 5 Grade 9 Mathematics: Radical Expressions

Question 5.
Joe and Ken were asked by their teacher to show how they write the given expression with rational exponents into a radical expression and simplify it. Below is the illustration of their solution.

Express the given expression with rational exponents into a radical expression then solve.

$(x^{12}y^{6})^{\frac{2}{3}}$

Joe's solution	Ken's solution
$= (x^{12}y^{6})^{\frac{2}{3}}$	$= (x^{12}y^{6})^{\frac{2}{3}}$
$= \sqrt{(x^{12}y^{6})^{3}}$	$= \sqrt[3]{(x^{12}y^{6})^{2}}$
$= \sqrt{x^{36}y^{18}}$	$= \sqrt[3]{x^{24}y^{12}}$
$= x^{18}y^{9}$	$= x^{8}y^{4}$

Question 5.
Describe the approach taken by each student **a1. Joe** and **a2. Ken** and indicate where any mistakes occurred

Question 5 a1. Joe

Your Answer:

Question 5 a2. Ken

Your Answer:

Item No. 006	Subject	Mathematics – Patterns and Algebra
	Grade Level	Grade 9
	Topic	Quadratic Equations
	PISA Competency	Recognizing functional relationships between quantities.
	K to 12 Curriculum Competency	M9AL-Ic-d-1: *The learner* solves equations transformable to quadratic equations (including rational algebraic equations). **Content Standards and Performance Standard:** • *The learner demonstrates understanding of key concepts of quadratic equations, inequalities and functions, and rational algebraic equations.* • *The learner is able to investigate thoroughly mathematical relationships in various situations, formulate real-life problems involving quadratic equations, inequalities and functions, and rational algebraic equations and solve them using a variety of strategies.*
	Higher Order Thinking Strategy adopted	✓ Explicitly using SOLO (from U → M → R) ☐ Alternative question/item taking a different approach, method, or technique ☐ Presenting information in a different form ☐ Reversing the way that a solution is traditionally achieved ✓ Providing an erroneous answer and asking students to correct mistakes ☐ Compare and contrast two similar features ☐ Other:

ACTIVITY 6 Grade 9 Mathematics: Quadratic Equations

Question 6. Terrence was tasked to solve a rational algebraic equation.

$$\frac{x+5}{x-3} + 3 = 3x$$

The following was his solution:

$\frac{x+5}{x-3} + 3 = 3x$	Step 1
$x + 5 + 3(x-3) = 3x(x-3)$	Step 2
$x + 5 + 3x - 9 = 3x^2 - 9x$	Step 3
$3x^2 - 13x - 9 = 0$	Step 4
$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(3)(-9)}}{2(3)}$	Step 5
$x = \frac{13 \pm \sqrt{169 - 108}}{6}$	Step 6
$x = \frac{13 \pm \sqrt{61}}{6}$	Step 7

Question 6 a1. Determine the mistakes on the solution. Explain why.

Your Answer:

Question 6 a2. Provide the correct solution and answer.

Your Answer:

Question 6 a3. If the question was to be scored out of ten marks. How many marks do you believe the student, Terrence, deserved for his solution?

Your Answer:

	Subject	Mathematics – Geometry
Item No. 007	**Grade Level**	Grade 9
	Topic	Trigonometry, Right Triangles
	PISA Competency	Identifying the mathematical aspects of a problem situated in a real-world context and identifying the significant variables.
	K to 12 Curriculum Competency	M9GE-IIIj-1: *The learner* solves problems that involve triangle similarity and right triangles. ***Content Standards and Performance Standard:*** • *The learner demonstrates understanding of key concepts of parallelograms and triangle similarity.* • *The learner is able to investigate, analyze, and solve problems involving parallelograms and triangle similarity through appropriate and accurate representation.*
	Higher Order Thinking Strategy adopted	✓ Explicitly using SOLO (from U → M → R) ✓ Alternative question/item taking a different approach, method, or technique ✓ Presenting information in a different form ☐ Reversing the way that a solution is traditionally achieved ☐ Providing an erroneous answer and asking students to correct mistakes ✓ Compare and contrast two similar features ✓ Other:

ACTIVITY 7 Grade 9 Mathematics: Geometry (Trigonometry, Right Triangles)

Question 7 a1. An observer in a lighthouse is 70 feet above sea level. He saw two boats in the water directly in front of him. The angle of depression to the nearest boat is $50°$. The angle of depression to the other boat is $25°$. Draw a diagram that summarizes the data provided.

Your Answer:

Question 7 a2. Using the information provided in **Question 7 a1.**, Determine the distance between two boats? **Hint:** use tangent ratios to help find the answer.

Your Answer:

Question 7 a3. Solve the same problem but use a different approach. Using the information provided in **Question 7 a1.**, Determine the distance between two boats? **Hint:** use the Sine Rule to help find the answer.

Your Answer:

Question 7 a4. What are the compare and contrast the two approaches used in **Question 7 a2.** using the tangent ratios and those employed in **Question 7 a3.** using the sine rule**.** What were the similarities and differences noticed?

Your Answer:

Similarities:

Differences:

Question 7 a5. Which approach did you prefer to use? What were the reasons for your views?

Your Answer:

Item No. 008	Subject	Mathematics – Patterns and Algebra
	Grade Level	Grade 9
	Topic	Inverse Variation
	PISA Competency	Recognizing functional relationships between quantities. Representing a situation mathematically, using appropriate variables, symbols, diagrams, and standard models.
	K to 12 Curriculum Competency	M9AL-llb-c-1: *The learner* solves problems involving variation. **Content Standards and Performance Standard:** • *The learner demonstrates understanding of key concepts of variation and radicals* • *The learner is able to formulate and solve accurately problems involving radicals.*
	Higher Order Thinking Strategy adopted	✓ Explicitly using SOLO (from U → M → R) ☐ Alternative question/item taking a different approach, method, or technique ☐ Presenting information in a different form ☐ Reversing the way that a solution is traditionally achieved ☐ Providing an erroneous answer and asking students to correct mistakes ☐ Compare and contrast two similar features ✓ Other: Extension possible beyond the question posed.

ACTIVITY 8 Grade 9 Mathematics: Inverse Variation

Question 8.
The average heart rates and the life span of some animals are shown in the following Table.

Mammal	Average Heart rate (beats/minute)	Average Life span (minutes)
Mouse	634	1 576 800
Rabbit	158	6 307 200
Lion	76	13 140 000
Horse	63	15 768 000
Elephant	?	?

Question 8 a1. Using information from the Table above: What is the average life span (in minutes) for the lion?

Your Answer:

Question 8 a2. Describe a feature that you can identify in the order of the list of mammals presented in the Table?

Your Answer:

Question 8 a3. Describe any pattern that is evident between the average size of the mammals and their average heart rate/minute?

Your Answer:

Question 8 a4. How does the average heart rate vary as the average life span?

Your Answer:

Question 8 a5. Assuming there is a basic inverse relationship between the average heart rate and the average life span of the mammals listed, develop an equation of the variation between average heart rate and average life span. Round off the constant to the nearest billion.

Your Answer:

Question 8 a6.

The average heart rate and the average life span of some animals are shown in the Table. An Elephant's life span is about 70 years. Estimate its average heart rate (in beats per minute). (Assume the average length of a year to be 365.25 days.)

Your Answer:

Grade 10

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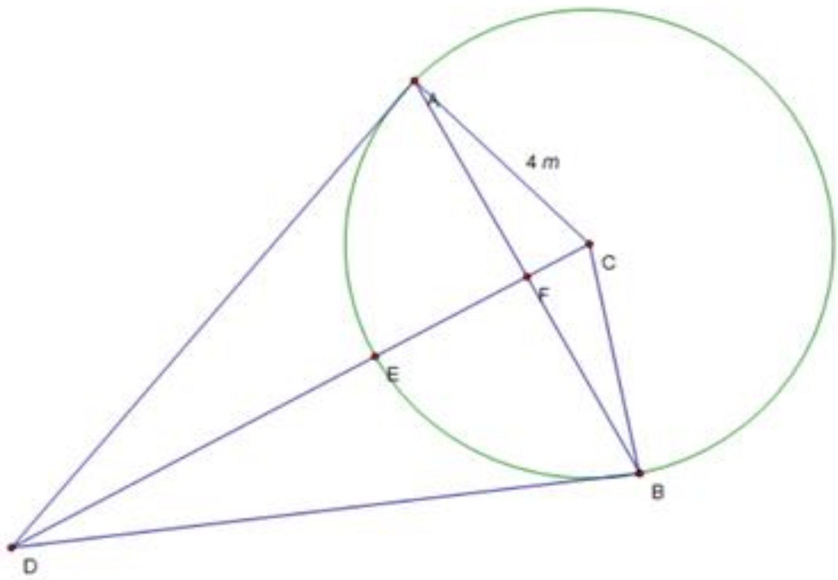
Item No. 001	Subject	Mathematics – Geometry
	Grade Level	Grade 10
	Topic	Circles
	PISA Competency	• Devising and implementing strategies for finding mathematical solutions • Employing Mathematical Concepts, Facts, Procedures and Reasoning.

	K to 12 Curriculum Competency	M10GE-IIf-2 : The learner solves problems on circles.
		
		Content Standards and Performance Standard:
		• ***The learner demonstrates understanding of key concepts of circles and coordinate geometry.***
		• ***The learner is able to***
		  • ***formulate and find solutions to challenging situations involving circles and other related terms in different disciplines through appropriate and accurate representations.***
		  • ***formulate and solve problems involving geometric figures on the rectangular coordinate plane with perseverance and accuracy.***

	Higher Order Thinking Strategy adopted	✓ Explicitly using SOLO (from U → M → R) ✓ Alternative question/item taking a different approach, method or technique ☐ Presenting information in a different form ☐ Reversing the way that a solution is traditionally achieved ☐ Providing an erroneous answer and asking students to correct mistakes ☐ Compare and contrast two similar features ☐ Other:

ACTIVITY 1 Grade 10 Mathematics: Circles

Question 1 Karl plans to construct a circular swimming pool with a radius of 4 meters (see the green circle below). Two sprinklers A and B are installed on the edge of the pool. A water source (Point D) is located 14 meters from the center of the pool (point C). Two lines of pipes, which is tangent to the pool, will be connected to the two sprinklers.



Question 1 a1. What is the measure of $\angle DAC$?

Your Answer:

Question 1 a2. How long is the pipe which connects the water source and sprinkler A, i.e., AD?

Your Answer:

Question 1 a3. How long is the pipe which connects the water source and sprinkler B, i.e., BD?

Your Answer:

Question 1 a4. Suppose the distance of two sprinklers is 7.65 m. Karl decided to construct a fountain at point F (midway between two sprinklers) and another pipe must be installed to connect it from the water source. How far is fountain from the water source?

Your Answer:

Question 1 a5. Find another (alternative) approach to solve this question of finding the length of DF?

Your Answer:

Item No. 002	Subject	Mathematics – Statistics and Probability
	Grade Level	Grade 10
	Topic	Permutations and Combinations
	PISA Competency	Recognizing functional relationships between quantities
	K to 12 Curriculum Competency	M10SP-IIId -e – 1: *The learner* solves problems involving permutations and combinations. **Content Standards and Performance Standard:** • The learner solves problems involving Permutations and Combinations • The learner is able to use precise counting technique and probability in formulating conclusions and making decisions.
	Higher Order Thinking Strategy adopted	✓ Explicitly using SOLO (from U → M) ✓ Alternative question/item taking a different approach, method, or technique ☐ Presenting information in a different form ☐ Reversing the way that a solution is traditionally achieved ☐ Providing an erroneous answer and asking students to correct mistakes ☐ Compare and contrast two similar features ☐ Other:

ACTIVITY 2 Grade 10 Mathematics: Permutations and Combinations

Question 2 a1. A basketball team consist of 5 players, although some additional players, called reserves, can be on the bench and used as replacements. By allocating alphabetical seven names (A, B, C, D, E, F, G) to the players, **list all possible** five-person team compositions that would be available to take the basketball court.

Your Answer:

Question 2 b1. Coach John has only 8 players for his basketball team because some members of the team are not available. To start a game, he needs to submit the names of his 5 players to the organizing officials. Determine how many different teams of five players can be selected from the available players.

Your Answer:

Question 2 b2. When the coach considered the eight players available, he noted that three players performed best when they took up a ‘guard’ position and another three performed best in the role of ‘power forwards’. The remaining two players had the necessary skills to take on the role of ‘center’. The coach decided that the team should comprise two ‘guards’ and two ‘power forwards’ and one ‘center’. To select the strongest team, the players would be selected based on their strengths. On these criteria: how many possible teams of five players can be selected?

Your Answer:

<table>
<tr><th rowspan="7">Item No. 003</th><th>Subject</th><th>Mathematics – Geometry</th></tr>
<tr><td>Grade Level</td><td>Grade 10</td></tr>
<tr><td>Topic</td><td>Circles (sector and arc length)</td></tr>
<tr><td>PISA Competency</td><td>Appreciating the power of abstraction and symbolic representation</td></tr>
<tr><td>K to 12 Curriculum Competency</td><td>M10GE-IIf-2: The learner solves problems on circles

Content Standards and Performance Standard:
• The learner demonstrates understanding of key concepts of circles and coordinate geometry.
• The learner is able to
  ○ formulate and find solutions to challenging situations involving circles and other related terms in different disciplines through appropriate and accurate representations.
  ○ formulate and solve problems involving geometric figures on the rectangular coordinate plane with perseverance and accuracy.</td></tr>
<tr><td>Higher Order Thinking Strategy adopted</td><td>✓ Explicitly using SOLO (from U → M)
☐ Alternative question/item taking a different approach, method, or technique
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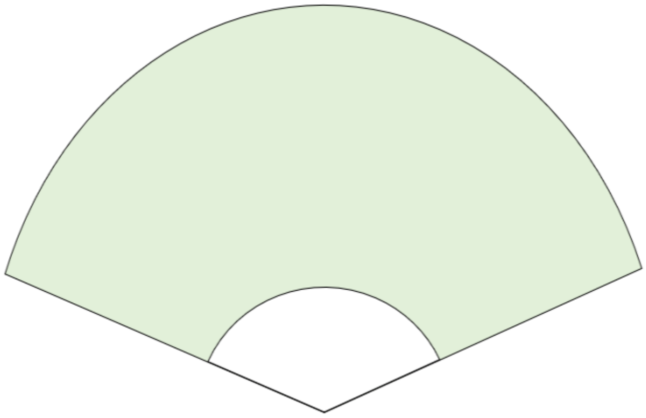
☐ Providing an erroneous answer and asking students to correct mistakes
☐ Compare and contrast two similar features
☐ Other:</td></tr>
</table>

ACTIVITY 3 Grade 10 Mathematics: Circles (Sector and Arc length)

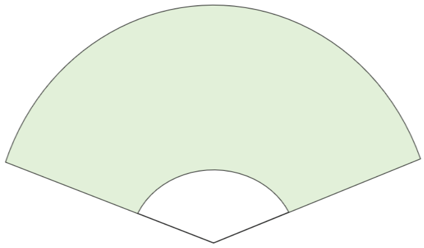
Question 3. Jane, a Grade 10 student, is making some folded fans made of 25-cm long strips of bamboo as fan ribs with its leaves covered with silk in green. The fan when fully opened has a central angle of 120°.



Question 3 a1. If the constructed fans will be used to decorate by making colorful circles calculate how many fans will be used to make a full circle?

Your Answer:

Question 3 a2. If the leaves of the fan are 5 cm away from the rivet, calculate the area of the fan covered by the silk cloth, the green shaded area.



Your Answer:

Question 3 a3. Jane wants to decorate the fan by adding laces, 2 cm wide, to the curved edge of the fan, $\frac{1}{2}$ cm of the lace is glued to the edge of the fan. Find the length of the lace she needs to decor the edge of the fan?

Your Answer:

Question 3 a4. Jane wants to decorate the fan by adding laces, 2 cm wide, to the curved edge of the fan, $\frac{1}{2}$ cm of the lace is glued to the edge of the fan. Calculate the area of the glued part with lace.

Your Answer:

<table>
<tr><td rowspan="6">Item No. 004</td><td>Subject</td><td>Mathematics – Statistics and Probability</td></tr>
<tr><td>Grade Level</td><td>Grade 10</td></tr>
<tr><td>Topic</td><td>Permutation and Combination</td></tr>
<tr><td>PISA Competency</td><td>Recognizing functional relationships between quantities.</td></tr>
<tr><td>K to 12 Curriculum Competency</td><td>M10SP-IIId-e-1: The learner solves problems involving permutations and combinations.

Content Standards and Performance Standard
• The learner demonstrates understanding of key concepts of combinatorics and probability.
• The learner is able to use precise counting technique and probability in formulating conclusions and making decisions.</td></tr>
<tr><td>Higher Order Thinking Strategy adopted</td><td>☐ Explicitly using SOLO (from M → R)
✓ Alternative question/item taking a different approach, method, or technique
☐ Presenting information in a different form
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✓ Providing an erroneous answer and asking students to correct mistakes
☐ Compare and contrast two similar features
☐ Other:</td></tr>
</table>

ACTIVITY 4 Grade 10 Mathematics: Permutation and Combination

Question 4 Alex is having trouble remembering his PIN on his Credit card. He recalls the digits he used would have been drawn from the following numbers 0, 2, 3, 4, 5, and 7. He also recalled that (i) the number he used was even; (ii) it did not start with zero; and (iii) it did not repeat any digits. His Bank in the Philippines uses only 4 digits as a PIN with a Credit card. He decided to work out the possible PINs that can be formed satisfying the above conditions. Here is his solution to the problem provided below.

Alex's Solution:
Repetition of the digits is not allowed, and it is an even number. A number is even if it ends with 0,2,4,6, and 8. Using the Fundamental counting Principle.

> **Step 1:** For the last digit considering the number is even, there are three choices. That is 0, 2, and 4.
> **Step 2:** Next to consider is the first digit, there will be 5 choices left since the number should not start with zero.
> **Step 3:** For the second digit, there will be 5 choices remaining since 0 can now be included in the selection.
> **Step 4:** For the third digit, there will be 4 choices.

Hence, the number of 4-digit even numbers is

$$5 \times 5 \times 4 \times 3 = 300$$

There are 300 possible PIN satisfying the above condition.

Question 4 a1. Unfortunately, the answer is not correct and contains a conceptual error. Find the step where the error occurred and describe why it is an error.

Your Answer:

Question 4 a2.
Provide the correct answer to the question.

Your Answer:

Question 4 a3.
If the question was to be scored out of ten marks. How many marks do you believe the student, Alex, deserved for his solution?

Your Answer:

Item No. 005	Subject	Mathematics – Geometry
	Grade Level	Grade 10
	Topic	Circle
	PISA Competency	Appreciating the power of abstraction and symbolic representation.

	K to 12 Curriculum Competency	M10GE-IIc-1: *The learner* derives inductively the relations among chords, arcs, central angles, and inscribed angles.
		
		Content Standards and Performance Standard:
		• *The learner demonstrates understanding of key concepts of circles and coordinate geometry.*
		• *The learner is able to*
		  ○ *formulate and find solutions to challenging situations involving circles and other related terms in different disciplines through appropriate and accurate representations.*
		  ○ *formulate and solve problems involving geometric figures on the rectangular coordinate plane with perseverance and accuracy.*

	Higher Order Thinking Strategy adopted	☐ Explicitly using SOLO (from U → M → R) ✓ Alternative question/item taking a different approach, method, or technique ✓ Presenting information in a different form ☐ Reversing the way that a solution is traditionally achieved ☐ Providing an erroneous answer and asking students to correct mistakes ☐ Compare and contrast two similar features ✓Other: Develop a concept map

ACTIVITY 5 Grade 10 Mathematics: Geometry (Circle)

Question 5 a1. The purpose of the activity is to create a concept map on solving a problem related to Circles. To begin, individual students should write down their own list of words that come to mind when we are solving a problem related to a circle.

Your Answer:

Question 5 a2. The purpose of this question part is to have look at the question of developing a concept map through Circle sub-topics made up by the students themselves. They can use the individual words to help them populate as many Circle sub-topics as they can.

Your Answer:

Question 5 a3. Groups of students should join and (i) share individual ideas with an explanation and (ii) create a combined list of Circle sub-topics to share with the class.

Your Answer:

(i) Agreed **Group** list

(ii) Agreed **Class** list

Question 5 a4.
Develop concepts maps. There are many possible maps. Some will be around individual Circle sub-topics and others will focus on seeing links among the Circle sub-topics.

Your Answer:

Item No. 006	Subject	Mathematics – Statistics and Probability
	Grade Level	Grade 10
	Topic	Permutations
	PISA Competency	Recognizing functional relationships between quantities
	K to 12 Curriculum Competency	M10SP-IIIb-1: *The learner* solves problems involving permutations. **Content Standards and Performance Standard:** • *The learner demonstrates understanding of key concepts of combinatorics and probability.* • *The learner is able to use precise counting technique and probability in formulating conclusions and making decisions.*
	Higher Order Thinking Strategy adopted	✓ Explicitly using SOLO (from U → M → R) ✓ Alternative question/item taking a different approach, method or technique ✓ Presenting information in a different form ☐ Reversing the way that a solution is traditionally achieved ☐ Providing an erroneous answer and asking students to correct mistakes ☐ Compare and contrast two similar features ☐ Other:

ACTIVITY 6 Grade 10 Mathematics: Permutations

Question 6 a1. Alexander and his 4 friends were scheduled to have their vaccination. As they went to the vaccination site, they were offered a long bench good for 5 people while waiting for their turn. In how many ways could Alexander and his friends to be seated on the bench?

Your Answer:

Question 6 a2. Suppose we still have Alexander and his 4 friends in the scene. In this case Alexander has a best friend among his four friends and these two people wanted to be seated next to each other. In how many ways could they be seated? (Show all working)

Your Answer:

Question 6 a3. Repeat the question (**Question 6 a2.)** above but consider the case where both Alexander and his friend are together but 2 of the friends **also** wish to sit together

Your Answer:

Question 6 a4. In this question all five friends are about to sit down on the bench with no special requests. **As it was about to happen** Alexander remembered he just had a difficult time with one of his four friends, and it might be best if they were **not** seated next to each other. In how many ways could the group be seated in the long bench?

Your Answer:

Question 6 a5. Can you think of your own scenario and determine the solution to your own problem? Write out the problem and the solution and share with a group in your class. See if the group agrees with your working and answer.

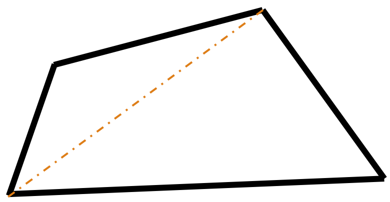
Your Answer:

Acceptable Answers to SOLO-based Items

(includes Writer's Reflections to provide background to readers)

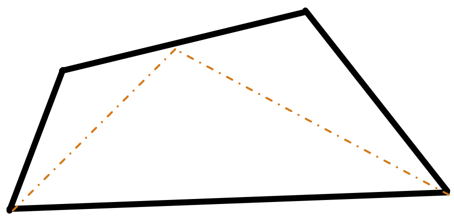
<table>
<tr><th colspan="2">Grade 7</th></tr>
<tr><th>Item no.</th><th>Answers</th></tr>
<tr><td></td><td>QUESTION 1 Grade 7 Mathematics</td></tr>
<tr><td>001</td><td>

Q1 a1:

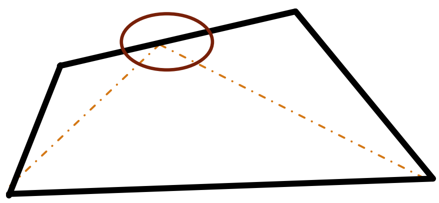


By drawing a diagonal line, the rectangle is divided exactly into two triangles. As the interior angle sum for one triangle is 180°. The angle sum of two triangles that make a quadrilateral will be 2x 180° = 360°

Q1 b1:



Q1 b2:



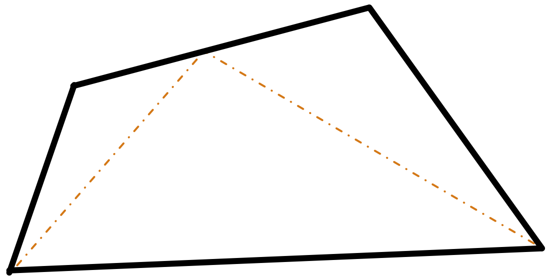
Three triangles would be formed when the triangles are formed from a side of a quadrilateral. See illustration above.

However, as 3 triangles in the interior are formed, students thinking might be perturbed into giving an answer of 180°× 3, which is wrong. This would allow students to realize that they need to subtract 180° as these were angles formed not in the interior angles in question, but along an edge/side of the quadrilateral.

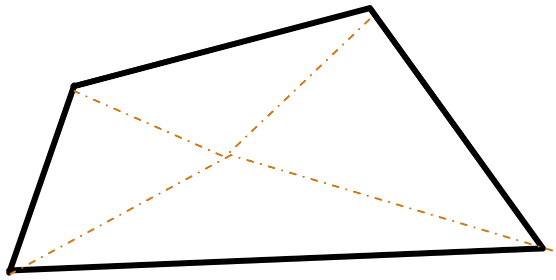
Q1 c1:
Here, the students would multiply 180° by 4, then subtract 360° as the angles formed in the center are irrelevant to the measure of the interior angles of the quadrilateral.

</td></tr>
</table>

For solutions where there are a number of 'unnecessary' angles, diagrams could just be deducted.



Q1 c2:



Students would be required to use three or more triangles to determine the angle sum of a quadrilateral with a number overlapping. This will require a systematic approach that will be different depending on the configuration of the triangles and the number of non-overlapping triangles involved.

Reflection:

In **Question 1 a1.** The teacher may just draw the quadrilateral. The quadrilateral must not be a square or a rectangle so that the respondents would not be fixated on the square and rectangle as representation of a quadrilateral. Respondents should be able to divide the quadrilateral into two non-overlapping triangles if they draw a diagonal from one corner (vertex) to the opposite corner (vertex). Check the respondents' language use with words such as corner, vertex, quadrilateral, diagonal, angle, etc. They should be able to spell these words as well.

The teacher should not draw the diagonal line first but allow respondents to explore how this question might be addressed. Respondents may need to be encouraged to think how they can use the information provided. This might help them to be able to think that the quadrilateral could be divided into two triangles by drawing a diagonal line from two opposite vertices, or by any other way.

The answers provided to the question in **Question 1 a1.** are designed to engage respondents on the task and establish baseline understanding. This question is a *unistructural* question in applying one simple idea to a familiar situation.

In Question 1 b1.

There are many ways of forming non-overlapping triangles inside the quadrilateral, though the number of triangles formed would also vary. Respondents should share their attempts with the class.

Now provide an alternative (another) correct answer/solution to the original activity. Taking a different approach, method or technique by dividing the quadrilateral into *more than two triangles*. How many triangles would you be able to draw inside the quadrilateral? How would these triangles be formed?

After using your new technique with more than two triangles: How would respondents compute for the sum of the interior angles of the quadrilateral?

In Question 1 b2.
Here the teacher asks respondents to consider cases where there are three triangles (or more) and have them described mathematically how they would explain their answer and share with other respondents in the class.

You will identify different solution strategies by forming (i) triangles by drawing a line that extends to another vertex to form triangles; (ii) triangles from a point along a side of the quadrilateral to vertices. Another strategy would be by identifying any point in the interior of the quadrilateral and extending lines from this point to all the vertices of the quadrilateral this is taken up in **Question 1 c1.**

Question 1 c1. This question concerns a triangle vertex within the quadrilateral. The main point here is for you, the teacher, to be able to discuss WITH respondents how the sum of the interior angles of the quadrilateral may be computed. Why do we need to subtract 180°? Or 360°?

A further extension (**Question 1 c2.**) would be to consider the case (if not done so already) where the triangles within the quadrilateral were overlapping. Respondents would be required to use three or more triangles to determine the angle sum of a quadrilateral. This question, if developed by the respondent, would be *relational*.

Be watchful of the responses of the respondents. Respondents might be able to present this solution first, so be flexible in orchestrating the discussions. Also, there could be many other approaches suggested by respondents such as forming too many triangles and by drawing many lines that form triangles.

Most of these questions are *multistructural* if you have already carried out extensive specific instruction. The reason is respondents are following a rule you have given. If the respondents are first presented with a question and asked to complete the question (problem solving) in some cases such as **Question 1 c1.** and **1 c2.** they would be *relational*.

	QUESTION 2 Grade 7 Mathematics

002

Q2 a1:

Polygon	Number of Sides	Smallest Number of non-overlapping triangles	Sum of Interior Angles
Quadrilateral	4	2	360°
Pentagon	5	3	540°
Hexagon	6	4	720°
Heptagon	7	5	900°
Octagon	8	6	1080°

Q2 b1:

(i) 12 sides means there will be 10 triangles. Hence, 10x180 = 1,800. Sum of interior angles is 1,800°

(ii) 30 sides means there will be 28 triangles. Hence, 28x180 = 5,040. Sum of interior angles is 5,040°

Q2 b2:

(i) 2,700/ 180= 15 which means there will be 15 internal non-overlapping triangles. Hence, the number of sides of the polygon will be 15 +2=17. Answer is 17 sides.

(ii) 7,560/ 180= 42 which means there will be 44 internal non-overlapping triangles. Hence, the number of sides of the polygon will be 42 +2=44. Answer is 44 sides.

Q2 b3:

The relationship (rule) that links the number of sides of a polygon to the angle sum of a polygon is

The angle sum of a polygon is equal to the number of non-overlapping triangles times 180 (the angle sum of a triangle) times the angle sum of a triangle. But the number of sides of the polygon is equal to the number of triangles plus 2. Therefore, the relationship between the Polygon angle sum and the number of sides can be written as

The angle sum of a polygon is equal to the number of sides of the polygon minus 2 times the angle sum of a triangle.

Q2 C1:

From the above question, Question 2 a1, we can see a rule emerging.

Let x be the interior angle sum for a polygon.
Let y be the number of sides of a polygon

Therefore,
The interior angle sum of a polygon = (number of non-overlapping triangles) times 180
The interior angle sum of a polygon = (number of sides of the polygon minus 2) times 180

$x = (y\text{-}2) \times 180$

Q2 c2:
- **(i)** 50 sides?
- **(ii)** 300 sides?

Q2 c3:
- **(i)** 8,640°
- **(ii)** 53,640°

Q2 c4:
The answer concerns the significance of obtaining the formula. While there is some thinking involved to get the formula, from then on it is a simple task of substation to get the answer saving a large amount of time and tedious effort.

Reflection:
It would be nice to organize the blackboard to show the works of the respondents based on several solution strategies. You may use a table to facilitate the consolidation of findings. Note that is a messy procedure as you, together with the respondents, would be working on the many and diverse solution strategies that would be coming from your students. Using a table or chart might help.

Note that diagonal solution is not possible for polygons with odd number of sides. Variety in solution paths are often unpredictable.

Here, the respondents would engage in comparisons of solution strategies. More so, they would need to come up with a formula to find the answer using the most convenient and intuitive solution for them. Let the respondents do the talking and the strategizing, perhaps in small groups and report back to the teacher. Give them time to think, collaborate, and communicate. The final questions are leading the respondents towards abstraction and its importance.

In terms of applying SOLO to Questions 2 a1, b1. and b2., the correct response can be considered *multistructural* as the questions require the respondent to follow a procedure and undertake a number of sequential steps. Questions 2 b3. and c1. require *relationa*l thinking. Using the formula that has been provided Question 2 c2. is *multistructural* once the rule has been established as it requires only successful substitution. Finally, Question 2 c3. is simply a case of substitution into the rule. Question 2 c4. can be answered at a number of levels and it depends on the depth and thinking behind the response to identify the level.

	QUESTION 3 Grade 7 Mathematics
003	**Q3 a1:** *School Year 2020* **Q3a2:** *a. False* *b. True* *c. True* *d. True* **Q3 a3:** Overall, the number of students enrolled increases except in the year 2019. The biggest percentage increase of enrolment from the previous year occurred in 2020. Grade 7 has the greatest number of students in each year, but in each year, there was a decline in student numbers for each year with the lowest number of students in each school year were in Grade 10. This question contains an initial Stem which has all the information required for the following three questions. The three questions represent a different SOLO level, namely unistructural, multistuctural and relational. The correct answer for: **Question 3 a1**. Is at the *unistructural* level as it requires one piece of information. **Question 3 a2.** The level is *multistructural* as the respondent is expected to undertake a number of single observations. **Question 3 a3**. The answer would be at the *relational* level as it is asking for some integration of responses in terms of overall response. Despite the aim of questions to be answered at different levels it is possible that respondents might give much lower levels than required for a complete response. For example, in the case of **Question 3 a3.** a respondent might provide only one fact about the data. If this was the case, then the response would be *unistructural* not the anticipated *relational* response.

	QUESTION 4 Grade 7 Mathematics
004	**Q4 a1:** The answer was not correct. The correct solution is 1 [(+81) ÷ (-9) + (-4)] – [(2) (-4) + (+6)] 2 [(-9) + (-4)] – [(2) (-4) + (+6)] 3 [(-9) + (-4)] – [(-8) + (+6)] 4 [-9 -4] – [-8 + 6] *6 added to -8 should be -2* 5 [-13] – [-2] 6 -13 + 2 7 = -11 **Q4 a2:** The success of the answer depends on the reason given. If there is only one reason the response would be coded as *unistructural*. An example might be: “I would give no marks because the student got the answer wrong.” A good response at *relational* level might be: “I would give 4 marks out of 5 because while the student got the wrong answer, the student has done much working that was correct. To get 5 the student would have had to have the working and answer correct. The student has shown a good understanding of order of operations, the use of brackets and its impact on undertaking various operations as well as basic operations using whole numbers.” **Reflection:** Question 4. Represents the Stem (overall information needed) for the following two questions. The first question, Question 4 a1, represents a SOLO level of relational as it expects respondents to go through the question looking for mistakes. The correct answer for Question 4 a1. is at the *relational* level as it requires an overview of the information provided. For Question 4 a2. This is more of a fun activity for the respondents, and it has a serious side as well as. The question offers the respondent a lot of practice at undertaking basic arithmetic operations and using order of operation information in a different way to what they would previously have experienced. However, it also helps respondents understand that showing your working out is beneficial as you can be rewarded by the marker for what you know not simply for getting the answer wrong. It also offers a non-threatening example for respondents to talk about mathematics and to defend their position. There is no correct answer to the question, but the quality of the discussion can have a SOLO level. The class could be split up into groups which gave the same score out of 5, and they could be asked to defend their decision to other groups in the class. For example, in the case of Question 4 a2. a respondent might provide only one fact about the data. If this was the case, the response would be *unistructural* not the anticipated *relational* response.

	QUESTION 5 Grade 7 Mathematics
005	**Q5 a1:** Nine (9) kilograms. 12 – 3 = 9 **Q5 a2:** Let *x* – be the number of ponkans Representation: $0.2x + 3 = 12$ $0.2x = 12 - 3$ $0.2x = 9$ $x = 9/0.2$ $x = 45$ ponkans can be placed in a full crate **Q5 a3:** Let *x* – be the weight of a ponkan Representation: $50x + 3 = 12$ $50x = 9$ $x = 9/50 = 0.18$ kilograms **Reflection:** **Question 5**. Represents Stem (overall information) needed for the following three questions. Respondents will need to return to the Stem to complete all question parts. **Question 5 a1**. is at the *unistructural* level as it requires **one** arithmetic operation (a subtraction involving two numbers given in the question). In addition respondents need to identify important data that are needed, and ignoring the data that are irrelevant, and having a plan/approach to solve the problem. For **Question 5 a2.** The level is *multistructural* as the respondent is expected to undertake a number of single sequential steps to reach a solution drawing on relevant pieces of information. This differs from the first question as it only required a single step or operation. In the case of a multistructural question the steps are in a known sequential order. For **Question 5 a3**. The answer would be at the *relational* level as the question does not follow a set of sequential steps. The question is asking respondents to reverse their train of thought from what has been asked previously. It requires a consideration (thinking) to consider all the information and then to work out what is needed. There needs to be oversight of the whole question and this needs to occur while individual steps are undertaken. In the case of a relational question the respondent must identify and maintain the connections among the constructs they presented.

	QUESTION 6 Grade 7 Mathematics
006	**Q6a1:** The range of scores is 89 – 76 = 13 **Q6a2:** Mean = $\frac{78 + 89 + 87 + 79 + 80 + 78 + 76 + 88}{8}$ Mean = 81.875 Therefore, Ruth's average test score in Mathematics is 81.875 **Q6a3:** Let x be the results of the 9th test in Mathematics 82.875 = $\frac{78 + 89 + 87 + 79 + 80 + 78 + 76 + 88 + x}{9}$ (82.875) (9) = 78 + 89 + 87 + 79 + 80 + 78 + 76 + 88 + x 745.875 - 78 - 89 - 87 - 79 - 80 - 78 - 76 - 88 = x *x* = 90.875 Therefore, Ruth needs to receive 90.875 for her next test. **Q6 a4:** 83.875 = $\frac{78 + 89 + 87 + 79 + 80 + 78 + 76 + 88 + x}{9}$ (83.875) (9) = 78 + 89 + 87 + 79 + 80 + 78 + 76 + 88 + x 745.875 - 78 - 89 - 87 - 79 - 80 - 78 - 76 - 88 = x *x* = 99.875 Therefore, Ruth can still receive an average of 83.875 on her Mathematics test scores but she needs to get 100% on her next test. **Reflection:** The four questions represent a different SOLO level. The correct answer for **Question 6 a1.** is at the *unistructural* level as it requires one piece of information through the use of a single operation. For **Question 6 a2.** The level is *multistructural* as the respondent is expected to undertake a number of single operations in a sequence, such as add all the scores and then undertake a division. For **Question 6 a3.** the answer would be at the *relational* level as it is asking for some integration of responses in terms of overall response and the approach needs to be considered and is more than following a series of known steps. For Q**uestion 6 a4.** The answer requires more than the overview of the situation. Respondents are expected to interrogate the data and look more deeply at possibilities.

	QUESTION 7 Grade 7
007	**Q7 a1:** The mode weight for the sample selected is **I52 grams.** It occurs four times. **Q7 a2:** **In order the weights are** 139, 145, 148, 148, 148, 149, 149, 149, **150**, **151**, 151, 152, 152, 152, 152, 153, 154, and 156. There are two middle scores 150 and 151. The median is the average of those two scores **150.5** **Q7 a3:** Observed mean: (139 + 145 + 148 + 148 + 148 + 149 + 149 + 149 + 150 + 151 + 151 + 152 + 152 + 152 + 152 + 153 + 154 + 156) / 18 ≈ 149.89 grams 152 - 148.89 = 2.11 The difference between the two weights is 2.11 grams. This means that the selected fruit was on average less than the advertised weight. **Q7 a4:** The advertised weight for 18 +1 =19 mangoes is 19 x 152 = 2,888 Let the weight of the additional mango be x The observed weight for 18 plus 1 selected is (139 + 145 + 148 + 148 + 148 + 149 + 149 + 149 + 150 + 151 + 151 + 152 + 152 + 152 + 152 + 153 + 154 + 156 + x) Hence 2888 - (139 + 145 + 148 + 148 + 148 + 149 + 149 + 149 + 150 + 151 + 151 + 152 + 152 + 152 + 152 + 153 + 154 + 156) = x x = 190 For the observed mean to be 152 grams, we need to add a mango weighing 190 grams which, while possible can be an outlier as it is far from the average mean of 148.89 grams. It would be highly unlikely to find such a large mango. **Q7 a5:** All measures of central contribute some information but in this case, we are most interested in the weight across a number of Mangoes. The mode tells us the most common weight of the mangoes. The median only tells us the position of the upper and lower 50% of the data values, it does not take all actual values into consideration. The arithmetic average or the mean takes all values into consideration and allows a direct comparison with average weight identified by the farmer. Therefore, it will be most appropriate to use the mean. **Reflection:** The five questions and these represent SOLO levels. They are based on the same Stem in **Question 7.** **Question 7 a1.** Asks for the mode value of the selected data. This is the score that occurs the most often. In this case the answer is 152 grams which occurred four time. It is unistructural as it requires a single answer being obtained by counting or observation of the data.

	For **Question 7 a2.** the level is *multistructural* as the respondent is expected to undertake a number of actions. The first involves writing the scores in ascending order. The next step is to find which score, or scores, is/are in the middle. In this case there are two middle scores. The median is the average of these scores. For **Question 7 a3**. the answer would be at the *multistructural* level as it is asking for a series of arithmetic actions in sequence. The question provides that all the scores need to be added and then divided by 18 to find the mean (average) of the weights and finally to check the difference between this figure and the advertised figure which will involve a subtraction. The number of actions needed to get the correct result may cause this question to be seen as more difficult by respondents than other multistructural questions. For **Question 7 a4.** the answer is relational as it is not a routine question. Respondent will need to understand the reason for all the individual steps required in obtaining a mean and establish how an increase in the mean can be accommodated and eventual work out a procedure to achieve a result. The correct answer for **Question 7 a5**. is potentially at the *unistructural* level as it requires one piece of information. However, the question is interesting as it requires a good understanding of the three measures of central tendency, all of which have value in certain questions. Depending on the quality of the answer and if the respondent explains why the mode and mean are not used this could easily require a relational response.

QUESTION 8 Grade 7

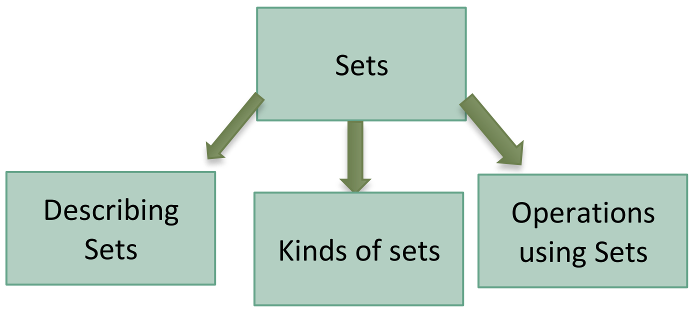
008

Q8 a1:
Individual list (probably a few words such as collection of things, commas, numbers or letters, words or objects, elements, members, brackets (braces)

Q8 a2:
Agreed Group list
Will include individual answers as well as ones that emerge as part of discussion
'Well defined' if all elements are listed, belonging to a set, not belonging to a set, finite set and infinite set, subset, empty set, null set, others?

Q8 a3:
Agreed Class list
This will bring together all the group responses. Also depending on where the class is at there may be an opportunity to talk about the union of sets and the intersection of sets and equal sets. If appropriate we can talk about the number of elements or cardinality but probably this is too far for Grade 7.

Q8 a4:
Possible start



Reflection:
The purpose of this activity is to use a series of lessons/topic/activity that has already been completed and ask the respondents to bring together the main ideas of the text/story/activity. The idea is to assist them organize these ideas in a logical way that will help their understanding and present these ideas using **a grade appropriate** and **age relevant** concept map.

One great value for respondents with the development of a concept map it provides a context in which respondents are expected to talk about mathematics, express their understandings or misunderstandings and listen to their peers as well as the teachers.

Most concept maps should have words on the arrows linking the main ideas. This way the different pathways represent a story.

Despite the aim of this activity to help respondents attain an overview to be answered at different levels it is possible that respondents may give much lower levels than required for a complete response. Nevertheless, the overall activity is about presenting a finished product which is at the *relational* level for the concept that is being addressed at the knowledge level required. Great care must be taken not to push the

	concept map into areas the respondents are yet to encounter. It is most likely that individual respondents will mainly be working at unistructural and multistructural SOLO levels. It will require encouragement and support to have a class end product on what they have already achieved at the relational level. For example, in the case of **Question 8 a1.** a respondent might provide only one fact about sets. If this was the case, then the response would be *unistructural* and it will be significant by the end of the activity that the respondent can provide a number of relevant ideas at the *multistructural* level.

Grade 8	
Item no.	**Answers**
	QUESTION 1 Grade 8 Mathematics
001	**Q1 a1:** Using the Triangle Inequality Theorem, let c be the measure of the third side. Let the 5 cm wooden block be the first side and 6 cm wooden block be the second side. Inequality 1: (1st side) + (2nd side) > 3rd side $5 + 6 > c$ $11 > c$ Inequality 2: (2nd side) + (3rd side) > 1st side $6 + c > 5$ $c > -1$ Inequality 3: (1st side) + (3rd side) > 2nd side $5 + c > 6$ $c > 1$ The third side must be greater than 1 cm but less than 11 cms. Using traditional notation, the answer would be $1 < c < 11$. **Q1 a2:** The intersection of Inequalities 2 and 3 is $c > 1$, which is Inequality 3. We could combine Inequalities 1 and 3 to come up with the possible integral values of c such that $1 < c < 11$. We organize the result by making a table:

1st side (a)	2nd side (b)	3rd side (c)	Perimeter= a+b+c
5	6	2	13
5	6	3	14
5	6	4	15
5	6	5	16
5	6	6	17
5	6	7	18
5	6	8	19
5	6	9	20
5	6	10	21

From the table developed we can see the smallest and largest perimeters that would be possible, namely, 13 cm is the shortest and 21 cm is the longest perimeter. Hence, there are 9 possible triangles whose perimeter is a whole number between 13 to 21 inches, inclusively.

Q1 b1:
Construct a table listing all the possible sides of the triangle whose perimeter is 10 and must satisfy the condition stated by Triangle Inequality Theorem.

Note: The order of the measurements is not important, hence, 1-4-5, 1-5-4, 4-1-5, 4-5-1, 5-4-1 and 5-1-4 refer to only one triangle.

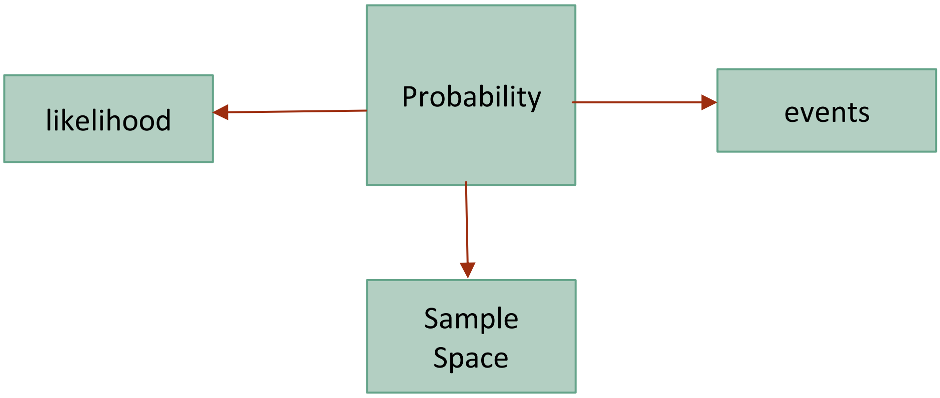
a	b	c	$a+b+c$	$a+b$	$b+c$	$a+c$	$a+b>c$	$b+c>a$	$a+c>b$
1	1	8	10	2	9	9	✗	✓	✓
1	2	7	10	3	9	8	✗	✓	✓
1	3	6	10	4	9	7	✗	✓	✓
1	4	5	10	5	9	6	✗	✓	✓
2	2	6	10	4	8	8	✗	✓	✓
2	3	5	10	5	8	7	✗	✓	✓
2	4	4	10	6	8	6	✓	✓	✓
3	4	3	10	7	7	6	✓	✓	✓

Therefore, Bernard has two possible triangles: a triangle whose measures are 2 in, 4 in and 4 in and a triangle whose measures are 3 cm, 3 cm, and 4 in.

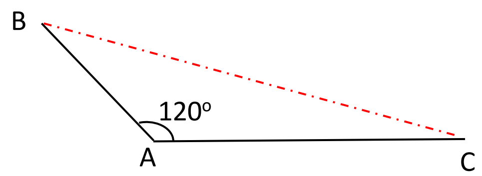
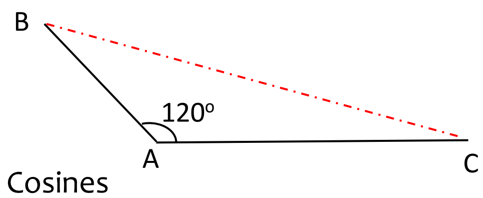
Question 1 a1. Concerns the application of a known procedure. To assist the respondents to start the question, the name of the theorem upon which the question is based is provided as well as what the theorem states. Those respondents who attain the side is greater than 1 is only part of the answer and their response would be coded as *multistructural* as they have undertaken a number of sequential steps. To achieve the correct answer, they need to look holistically at the question and indicate the lower **and upper** boundaries. If respondents do this their response is at the *relational* level.

Question 1 a2. The answer to this question requires respondents to use the information from the previous question. The answer can be found by tabulating the information. This would require interpreting the statement about *c* $1 < c < 11$ and linking this with the values for *a* and *b*. Normally this would be considered a *multistructural* activity but the need to manage *a*, *b*, and *c* together might mean it is *relational*.

	In both cases respondents will be shown to use different approaches to solving the question. These different approaches should be shown to the class and discussed as to what is different and what is the most efficient. **Question 1 b1.** This is a *relational* question. The respondent is provided with the answer for the perimeter and is asked to find the possible sides of the triangular mold is all lengths are integers. This is a reversible question and requires some patience on the respondent and constant checking.

	QUESTION 2 Grade 8 Mathematics
002	**Q2 a1:** Individual list, probably a few words such as: Probability, likelihood, events, unlikely, likely, certain, 50-50 chance, event, sample space, expected outcomes, impossible outcomes, possible outcomes, certainty, biased, unbiased, probability range, random experiment, equally likely events, probability of an event, possible outcome, total number of outcomes, experimental probability, theoretical probability. **Q2 a2:** Agreed Group list Will include individual answers as well as ones that emerge as part of discussion **Q2 a3:** Agreed Class list This will bring together all the group responses. Also depending on where the class is at there may be an opportunity to talk about specific simple examples such as throwing a die, or tossing a coin, or drawing colored beads from a bag. If appropriate students can talk about the number of elements in a sample space, and how this is critical for probability understanding. **Q2 a4**  **Reflection:** The purpose of this activity is to use a series of lessons/topic/activity that has already been completed and ask the respondents to bring together the main ideas of the text/story/activity. The idea is to assist them organize these ideas in a logical way that will help their understanding and present these ideas using **a Grade appropriate** and **age relevant** concept map. One great value for respondents with the development of a concept map it provides a context in which respondents are expected to talk about mathematics, express their understandings or misunderstandings and listen to, and learn from, their peers as well as the teachers. Most concept maps should have words on the arrows linking the main ideas. This way the different pathways represent a story.

	Despite the aim of this activity to help respondents attain an overview to be answered at different levels it is possible that respondents may give much lower levels than required for a complete response. Nevertheless, the overall activity is about presenting a finished product which is at the *relational* level for the concept that is being addressed at the knowledge level required. Great care must be taken not to push the concept map into areas the respondents are yet to encounter. It is most likely that individual respondents will mainly be working at *unistructural* and *multistructural* SOLO levels. It will require encouragement and support to have a class end product on what they have already achieved at the *relational* level. For example, in the case of **Question 2 a1.** a respondent might provide only one fact about probability. If this was the case, then the response would be *unistructura*l and it will be significant by the end of the activity that the respondent can provide and talk about a number of relevant ideas at the *multistructural* level.

Grade 9	
Item no.	Answers
	QUESTION 1 Grade 9 Mathematics
001	**Q1 a1:** From Town A to Town B Average speed S_{AB} is (60 kph) Time taken is 45 minutes (3/4 hour) D_{AB} = S_{AB} x T_{AB} D_{AB} = 60 x ¾ = 45 km From Town A to Town C = (90 kph) (3/4 hr) D_{AC} = 67.5 km The total distance traveled by the two friends is 45 km + 67.5 km = 112.5 km **Q1 a2:**  **Q1 a3:** From Question 21 1a. we found From Town A to Town B, D_{AB} = 45 km From Town A to Town, D_{AC} = 67.5 km The two roads form an angle of 120°. The following diagram summarises this information.  Applying the Law of Cosines $a^2 = c^2 + b^2 - 2bc\cos 120^0$

$$a^2 = 45^2 + 67.5^2 - 2(45)(67.5)(-\frac{1}{2})$$
$$a^2 = 2{,}025 + 4{,}556.25 + 3{,}037.5$$
$$a = 98.08$$

There for the distance between Town C and Town B, D_{BC} is 98.08 km

Reflection:
The three of the four questions of **Question 1.** represent the SOLO level of *multistructural*. Only question, **Question 1 a2.,** is at the *unistructural* level, as it requires translating a straightforward description into a diagram.

The other questions all require a sequential predictable series of actions resulting in the correct answer.

Of importance is the fact that questions rely on the answer to a previous question. This answer to the previous question does not have to be completed a second time, but can be referred to in the answer. It is enough to state where the answer came from and to use it as a piece of information to help address the following question.

Using previous data from a question belonging to the same base information is often common in math and students need to be informed of the possibility. It is not cheating to use work on a question that has previously been the focus of a question.

	QUESTION 2 Grade 9 Mathematics
002	**Q2 a1:** ∠DAE = ∠BAC because $\Delta BCA\ and\ \Delta DEA$ share a common $\angle A$ ∠BCA = ∠DEA because they are both right angles. $\angle ADE = \angle ABC$, as the angle sum of both triangles is 180° Therefore $\Delta BCA\ and\ \Delta DEA$ are similar as the corresponding angles of both triangles are equal. **Q2 a2:** ΔDEA is an isosceles triangle since the length of the stick is the same as Ivan's arm. Because ΔBCA is similar to $\Delta DEA,$ then ΔDEA is isosceles as well. Therefore, AC = CB = 161 inches. So the height of the tree is 161 inches + 61 inches (the height to Ivan's shoulder) = 222 inches. There are 2.54 cm in 1 inch so the height of the tree in meters is (222 x 2.54)/ 100 $m \approx 5.64\ m$ **Q2 a3:** The distance of Ivan from the tree is equal to the height of the tree (121 inches) minus the height of Ivan's shoulder (61 inches) = 121 – 61 = 60 inches Because of the fact that $\Delta BCA\ is\ similar\ to\ \Delta DEA$, and both triangles are isosceles. As before 1 inch = 2.54 cm Therefore (60 in x 2.54)/100 = 1.524 meters **Reflection:** There are 3 questions addressing this trigonometric question based on a real-world context involving a piece of wood of a special length equal to the length of the extended arm (as far as his fist extended) of Ivan. Also, the height of the stick above the ground is up to Ivan's shoulder, not his head. The basis of the use of this technique is on identifying similar triangles which in this case are also isosceles triangles. **Question 2 a1.** Concerns establishing by a proof that two triangles as similar. A property of similar triangles is that all corresponding angles are equal or the equivalent idea since all triangle angle sum is the same then we only require that two corresponding angles are equal. This question is a standard geometric question, and it is a the multistructural level. The correct answer for **Question 2 a2**. Also requires that the triangles are observed to be isosceles with the base and the perpendicular height being equal. Again, this require a small number of observations and the correct result is at the *multistructural* level as it requires more than one piece of discrete information. For **Question 2 a3.** the height of the tree is known, and the student is required to work

	in reverse as compared to **Question 2 a2.** The level here is *relational* as the student is expected to unpack what had previously been undertaken and reformulate it in a new but integrated way.

	QUESTION 3 Grade 9 Mathematics
003	**Q3 a1:** $5x^2 - 30x + 12$ $= 5(x^2 - 6x + 9 - 9) + 12$ $= 5(x - 3)^2 - 45 + 12$ $= 5(x-3)^2 - 33$ Therefore $a = 5$; $h = 3$; $k = -33$ **Q3 a2:** Area of rectangle is $3y \times (y + 4) = 3y^2 + 12y$ Therefore $3y^2 + 12y = 36$ $3y^2 + 12y - 36 = 0$ $3(y^2 + 4y + 4 - 4) - 36 = 0$ $3(y + 2)^2 - 12 - 36 = 0$ $3(y + 2)^2 - 48 = 0$ The answer as required. **Q3 a2:** Solving the equation $3(y + 2)^2 - 48 = 0$ $(y + 2)^2 = 16$ $y + 2 = +4$ or -4 $y = 2$ or -6 The lengths of the rectangle are $3y$ and $y+4$. On substitution of y values this is 6 and 6 or –18 and –2 As the sides of a rectangle cannot have negative lengths then the only practical value for y is 2. Thus, the sides are all equal to 6 cm. This means that the rectangle is a square with a perimeter of 24 cm. **Reflection:** These questions are concerned with developing and using the completing the square approach to quadratic expressions. **Question 3 a1**. Is a relative straight forward approach to establish if the students can develop a quadratic expression in terms of a binomial that is squared. The level is *multistructural* as the respondent is expected to undertake a number of familiar steps in sequence ensuring that the expression does not change in value. **Question 3 a2**. This question is about using the completing the square technique to confirm the answer given in the question. The prompt to use this approach can be taken from **Question 3 a1.** and/or from the general look of the answer. The approach to obtain the answer involves a predictable sequence of steps. Again, the level of this question is *multistructural*. For **Question 3 a3**. the answer would be at the *relational* level as it is asking for some

	integration of responses in terms of overall response. Respondents need to work backwards from the idea of perimeter and to sort out how to achieve the solution. Note that the solution should explain why the values of y that lead to a negative answer are not appropriate in this case and need to be excluded. Note also, that the figure turns out to be a square in this question. Need to confirm with respondents that a square belongs to the family of rectangles. A square is a rectangle with adjacent sides equal. Some students might think in error that the adjacent sides of a rectangle are not equal. The name of a shape with this characteristic is referred to as an oblong not a rectangle.

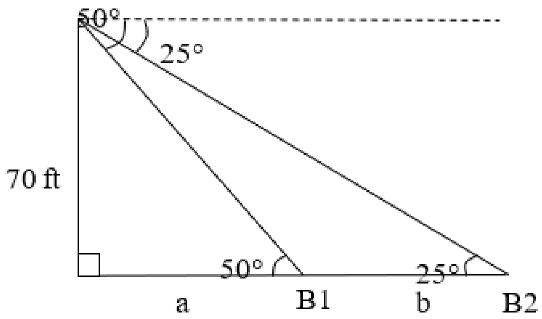
	QUESTION 4 Grade 9 Mathematics
004	**Q4 a1:** Jose made a mistake in Step 4 because he factored the left side of the equation using the "plus sign" instead of the negative sign of the binomial. This explains why the rest of the steps may also be wrong. **Q4 a2:** STEP 4: $(x-\frac{7}{6})^2=\frac{121}{36}$ STEP 5: $\sqrt{(x-\frac{7}{6})^2}=\pm\sqrt{\frac{121}{36}}$ $x-\frac{7}{6}=\pm\frac{11}{6}$ STEP 6: $x=\frac{7}{6}\pm\frac{11}{6}$ $x=\frac{7}{6}+\frac{11}{6};\ x=\frac{7}{6}-\frac{11}{6}$ Jose's final answer should be: $x=\frac{18}{6}=3,\ x=\frac{-4}{6}=\frac{-2}{3}$ **Q4 a3:** Teacher's judgment call NOTE: The reasons should focus on giving recognition to what the student has been able to achieve. How much was correct, not just counting steps. The number and seriousness of the mistakes. If one mistake is made and the following working using that mistake is correct, then the following work cannot be considered wrong mathematics. The only provisory here is if the mistake makes the question simpler. **Reflection:** **Question 4** offers a solution by student, Jose, to the solution of a quadratic equation using a specific method, namely, completing the square. **Question 4 a1**. Asks for the first error in the solution attempt by Jose. This is at the multistructural level as the respondent needs to consider each step sequentially. **For Question 4 a2**. the level is *multistructural* as the respondent is expected to undertake a number of single, sequential steps correctly as they proceed through a typical solution process. **For Question 4 a3**. the answer would be at the *relational* level as it is asking for some integration of responses in terms of overall response to the mark awarded to Jose. However, the respondents may provide different quality answers. For example, a respondent might provide only one fact about the data. If this was the case, then the response would be *unistructural* not the anticipated *relational* response.

	QUESTION 5 Grade 9 Mathematics
005	**Q5 a1:** **Joe** was incorrect because he changed the given expression into radicals by taking the numerator as the index of the radical and taking the denominator as the exponent of the radicand. In **line 2** Joe has found the square root cubed where it should be the cube root squared. After making this error the following working is consistent with the mistake made. **Q5 a2:** Ken was correct and there are no errors in his work, because he changed the given expression into radicals by taking the denominator of the fractional exponent as the index of the radical and taking the numerator as the exponent of the radicand. **Reflection:** This is a similar activity that has been offered in some activities, but it is organized slightly differently in this set. The respondent is provided with two choices concerning a radical expression that two students have been asked to simplify. As it turns out, one approach is correct, and one is incorrect. In future questions of this type it is possible for there to be different errors in both responses. For **Question 5 a1**. and **Question 5 a2.** the level is *multistructural* as the student is expected to work their way sequentially through each step themselves is this correct or not.

	QUESTION 6 Grade 9 Mathematics
006	**Q6 a1:** There were two errors. The first error starts at Step 4. After he multiplied all the terms by the LCD $x-3$(Line 2) and applied distributive property (Line 3), unfortunately, when similar terms were combined, the constant term 5 was overlooked (Line 4). $$x+5+3x-9=3x^2-9x$$ It should be $4x-4=3x^2-9x$, instead of $4x-9=3x^2-9x$. The second error occurs at Step 6 when the student correctly multiplied 4 x 3 x 9 but did not correctly take into consideration of the two negative signs. $x=\frac{13\pm\sqrt{169+108}}{6}$ Step 6 **Q6 a2:** The correct solution is: $4x-4=3x^2-9x$ $3x^2-13x+4=0$ $(3x-1)(x-4)=0$ $3x-1=0\,;x-4=0$ $x=\frac{1}{3}\,;x=4$ So, the correct answer is $x=\frac{1}{3}$ or $x=4$ **Q6 a3:** Teacher's judgment call NOTE: The reasons should focus on giving recognition to what the student has been able to achieve. How much was correct, not just counting steps. The number and seriousness of the mistakes. If one mistake is made and the following working using that mistake is correct, then the following work cannot be considered wrong mathematics. The only provisory here is if the mistake makes the question simpler. **Reflection:** **Question 6 a1. Is** about the solution of a quadratic equation by a student. There are two mistakes not one. Because the solution needed is a number of sequential steps to be performed correctly, finding one mistake would be coded as unistructural and the two mistakes would be at the multistructural level. The second question, **Question 6 a2.,** requires a *multistructural* response as it requires a set of correct procedures performed in sequence, composed of a predictable number of small steps.

	The final question, **Question 6 a3.** can offer several levels of response. However, the highest level, *relational*, will require the respondent to first identify both the mistakes and the answer and find the balance of what has been achieved and what has not. This can be achieved by teasing out the magnitude of the error as compared to the amount of information and processes the respondent demonstrated that they understood. Interestingly, the student made the question harder by making a mistake and needing to employ the quadratic formula, which was known correctly, although while the student was able to make the correct substitution, he failed in simplifying the arithmetic needed under the square root sign.

	QUESTION 7 Grade 9 Mathematics
007	**Q7 a1:**



Q7 a2:

Let *a* be the distance of boat B_1 from the base of the lighthouse

Let *b* be the distance between two boats B_1 and B_2

Using Tangent ratios

Using trigonometric ratios we have, $tan\ 50° = \frac{70}{a}\ and\ tan\ 25° = \frac{70}{a+b}$

$$a = \frac{70}{tan50°}\ ;\ a + b = \frac{70}{tan25°}$$

$$a = \frac{70}{tan25°} - b$$

Since both equations are equal to a, equate them

$$\frac{70}{tan50°} = \frac{70}{tan25°} - b$$

$$b + \frac{70}{tan50°} = \frac{70}{tan25°}$$

$$b = \frac{70}{tan25°} - \frac{70}{tan50°}$$

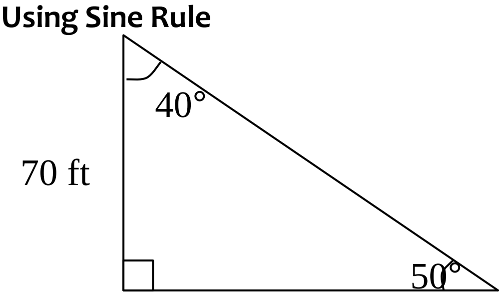
$$b \approx 91.38\ feet$$

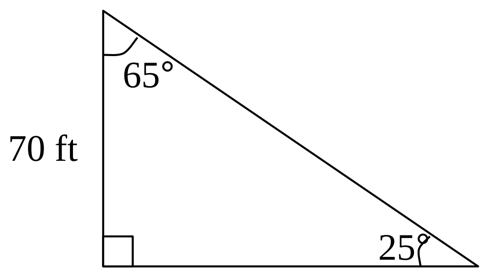
Therefore, the distance of the two boats is approximately 91.38 feet

Q7 a3:

Let *b* be the distance between two boats

Using Sine Rule





$$\frac{a}{sin40°}=\frac{70}{sin50°}\ ;\ \frac{a+b}{sin65°}=\frac{70}{sin25°}$$

$$a=\frac{70sin40°}{sin50°}\ ;\ a+b=\frac{70sin65°}{sin25°}$$

$$a=\frac{70sin40°}{sin50°}\ ;\ a=\frac{70sin65°}{sin25°}-b$$

Since both equations are equal to a, equate them

$$\frac{70sin40°}{sin50°}=\frac{70sin65°}{sin25°}-b$$

$$b+\frac{70sin40°}{sin50°}=\frac{70sin65°}{sin25°}$$

$$b=\frac{70sin65°}{sin25°}-\frac{70sin40°}{sin50°}$$

$$b\approx 91.38\ feet$$

Therefore, the distance of the two boats is approximately 91.38 feet

Q7 a4: Teacher's judgment call
Similarities
(list here)

Have the students complete this on their own first, for both similarities and differences. Then when marking establish a list on the board where all students contribute. Then discuss each one with the students and discard any that seem inappropriate or not mathematical. Then have students mark their own from the summary list on the board.

Differences
(list here)

Q7 a5: Teacher's judgment call
Preference here
(List preference)

Reasons here
Have the students complete this on their own first, with their reasons. Then when marking establish a list of reasons on the board where all students contribute. Then discuss each one with the students and discard any that seem inappropriate or not mathematical. Then have students mark their own from the summary list on the board.

Reflection:
Question 7 a1. Has been isolated as an individual question. It is important that respondents are given opportunities to independently draw a diagram to summarize given data in a question as opposed to being given the diagram. Also, it means if

	respondents have trouble with this aspect they can be supported. It also means that after the diagram is corrected all respondents can start on a level playing field in order to proceed. The SOLO codes for this question differ from the ones that follow. In general, the correct answer to this question is at the *multistructural* level where all only relevant data are provided in the question, and this is all included in the diagram. **Question 7 a2.** And **Question 7 a3.** Are *multistructural* questions as they are about applying a known formula and substituting know variables into the equations. These should be corrected before moving onto the real purpose of this activity which are the following two questions. This is not to say that these questions are not important. It is a critical skill to find the appropriate approach to use and to carry out the procedures successfully. However, this can all be done without any conceptual thinking about what has happened. **Question 7 a4.** And **Question 7 a5.** Require students to look at the work they have completed and to analyze it in two ways. These two questions have the potential to be relational depending on the respondent answers. If the respondent focuses only on one idea in either question the answer will be at the *unistructural* level, a number of individual ideas would be at the *multistructural* level and a number of ideas that are integrated to provide a response would be *relational*.

	QUESTION 8 Grade 9 Mathematics
008	**Q8 a1:** The average life span for a lion is 13, 140, 000. **Q8 a2:** The list shows mammals that are increasing in size. The mouse is the smallest and the elephant is the largest. The other animals are in order between these two. **Q8 a3:** As the average size of the mammal gets bigger the average number of heart beats per minute becomes smaller. **Q8 a4:** As the average heart rate gets lower, the average life span increases. There is an inverse variation between the average heart rate of mammals and their average life span. **Q8 a5:** Let *y* be the life span and *x* be the heart rate $y = \frac{k}{x}$ $1\,576\,800 = \frac{k}{158}$ k = (1 576 800) x (158) k = 2 000 000 000 (correct to the nearest billion) Therefore, the equation of variation is $y = \frac{2\,000\,000\,000}{x}$ **Q8 a6:** An average elephant's life span in minutes is approximately 70 (years) x 365.25 (days) x 24 (hours) x 60 (minutes) 36 817 200 minutes Using for formula developed in Question 28 a3. The equation of variation is $y = \frac{2\,000\,000\,000}{x}$ $36\,817\,200 = \frac{2\,000\,000\,000}{x}$ $x = \frac{2\,000\,000\,000}{36\,817\,200}$ $x \approx 54.32$ (correct to 2 decimal places) The average heart rate for an elephant is expected to be approximately 54.32 beats/minute. **Reflection:** The correct answer for **Question 8 a1**. And **Question 8 a2.** Are questions for a first cycle SOLO response. It requires information directly from the table. This questions, while very straightforward, encourages the respondent to look at the table carefully and find

relevant pieces of information. It also encourages the respondent to start the question.

For **Question 8 a3**. The level is unistructural as the respondent is asked to find a single pattern in one column of the Table. This is the start of what might be a mathematical response to the question.

Question 8 a4. The level is unistructural as the student is prompted to consider a simple pattern in two individual rows and then make a consideration of some common link.

For **Question 8 a4**. The answer would be at the multistructural level as it is asking for the respondent to adopt a mathematical approach by noting the 'inverse' relationship that is evident and maybe writing down the formula.

In **Question 8 a5**. The answer would be at the multistructural level as it is asking for substitution of data into a known equation and solving that equation.

In **Question 8 a6**. The correct answer would be at the relational level. This involves using information generated in earlier parts of the question, some establishing that the units used are appropriate before substitution into an equation and solving the equation.

It would be worth a discussion and exploration by the class (i) what the word 'average' means in this context? And (ii) why the word 'average' is used here? Also for discussion is round decimals and the need for, or use of, this.

At the end of the activity the class might discuss how humans might fit into this information and the implications that are possible.

	Grade 10
Item no.	Answers

	QUESTION 1 Grade 10 Mathematics
001	**Q1 a1:** Since the tangent of the circle is always perpendicular to the radius of the circle at the point of tangency, and perpendicular segments intersect at a right angle, then $\angle DAC = 90o$. **Q1 a2:** Since triangle CAD is a right triangle with $\angle ACB$ as the right angle, and $\underline{CD}$ as the hypotenuse, then by Pythagorean Theorem, $AD^2 = CD^2 - CA^2$ $= 14^2 - 4^2$ $AD^2 = 180$ $AD = \sqrt{180} = 13.416m$ **Q1 a3:** Triangles ADC and BDC are congruent as both have a right angle, equal hypotenuse length and one other side of both triangles is also equal. CD (hypotenuse) is common to both triangles, and AC = BC has both are radii of the same circle. Therefore $AD = BD$, hence $BD = AD = \sqrt{180} = 13.416m$. **Q1 a4:** Because triangles ACD and BCD are congruent this means that $\angle ACF = \angle BCF$ Therefore, as triangle ACB is isosceles CF bisects AB at right angles. This also means that CE which contains points C and F is also perpendicular to AB Since F is the midpoint of $\underline{AB}$, then $AF = \frac{7.65}{2} = 3.825m$. Also, $\underline{CE}$ is perpendicular to $\underline{AB}$ by theorem. Hence, $\angle AFD = 90o$. By Pythagorean Theorem, $AD^2 = AF^2 + DF^2$ $DF = \sqrt{AD^2 - AF^2}$ $= \sqrt{13.416^2 - 3.825^2}$ $= 12.859m$ The fountain is 12.859 meters away from the water source. **Q1 a5:** Triangle CFA is a right triangle with $\angle CFA = 90$, hence, by Pythagorean Theorem, $AC^2 = AF^2 + CF^2$

$$\begin{aligned} CF &= \sqrt{AC^2 - AF^2} \\ &= \sqrt{4^2 - 3.825^2} \\ &= 1.170m \end{aligned}$$

Since $DC = DF + CF$ then, $DF = DC - CF$

$$\begin{aligned} &= 14 - 1.170 \\ &= 12.83m \end{aligned}$$

It is noted that the two solutions are the same up to one decimal place only because of rounding off error.

Reflection:
The five questions each represent a particular SOLO level. The correct answer for **Question 1 a1**. is at the *unistructural* level as it requires one piece of information from a known theorem. Also, although not reliable, the visual image of the answer looks like the real answer as well. Hence it would be valuable to check with the respondent the reason for their correct answer. If the answer is based only on visual criteria, then discussion will be needed as this is not an appropriate response for a geometry question as often diagrams are used to support the solution process and not drawn to scale accurately.

For **Question 1 a2.** the level is *multistructural* as the respondent is expected to use the Pythagoras theorem in a non-prompted situation involving given data.

For **Question 1 a3**. the level is also *multistructural* for similar reasons. In this question the respondent is expected to use the right-angle congruency theorem in a non-prompted situation involving given data.

For **Question 1 a4.** the level is *relational*. Again, it is important that respondents do not assume answers based on visual appearance. For example, there needs to be thought given to why point F is the midpoint of AB, and not rely on the fact that it looks like it.

Question 1 a5. is also at the *relational* level as the respondent must seek out an alternative solution having already found a solution. This is not a particularly easy thing to do as the earlier approach will still be evident in the thinking of the respondent.

Also, in the case of **Question 1 a5.** the impact of rounding error is worth exploring in detail and discussing.

	QUESTION 2 Grade 10 Mathematics
002	**Q2 a1:** (ABCDE) (ABCDF) (ABCDG) etc should be looking to see some systematic way that the list is being developed to ensure all combinations (*n*=21) have been found. **Q2 b1:** The problem is about selecting the 5 players among 8 available members of the team to play in the basketball game. Since the order is not important in the selection of players, it is better to use combination instead of permutation. From the situation, we will select 5 players from 8 available players. $C(n,r) = \frac{n!}{(n-r)!\,r!}$ $n = 8;\ r = 5$ $C(8,5) = \frac{8!}{(8-5)!\,5!}$ $= \frac{8 \text{ x } 7 \text{ x } 6 \text{ x } 5!}{3!\,5!}$ $= 8 \text{ x } 7$ There are 56 ways of selecting 5 players from 8 available players. **Q2 b2:** Coach John will field 2 guards, 2 power forwards, and 1 center. By Fundamental Counting Principle: $C(2,1) \text{ x } C(3,2) \text{ x } C(3,2)$ $= \frac{2!}{1!\,1!} \text{ x } \frac{3!}{1!\,2!} \text{ x } \frac{3!}{1!\,2!}$ $= 2 \text{ x } 3 \text{ x } 3$ $= 18\ ways\ of\ selecting\ 5\ players.$ **Reflection:** The three questions represent a different SOLO level. This first question was an opportunity, which was valuable background to for **Question 2 b1**. to check the 'validity' of the rule by writing down all the possible teams. Respondents were expected to use some systematic pattern and possibly verifying it through application of the known combination rule. This question was separated from the following two questions as it was felt the size of the numbers in **Question 2 b1.** and **Question 20 b2.** made it too tedious to list so many different possible teams.

	Despite the general area of permutations and combinations to be perceived to be challenging to obtain the correct answer, **Question 2 b1.** is at the *unistructural* level (in the second cycle) as it requires the application of a known rule in a single straight-forward application. For **Question 2 b2.** the level is *multistructural* as the respondent is expected to undertake a number of single observations in sequence by applying known rule of three occasions.

	QUESTION 3 Grade 10 Mathematics
003	**Q3 a1:** 3 fans could be made as the angle at the center is 360° **Q3 a2:** Area of a circle: $A = \pi r^2$ $= \pi(25)^2$ $= 625\,\pi$ $= (625)(3.14)$ $= 1962.5\ sq\ cm^2$ Area of the fan: The area of the fan which is the sector of the circle in the given problem is $\frac{1}{3}$ of the area of the circle. $A_F = \frac{1}{3}(\pi)(25)^2$ $A_F = 654.17\ sq\ cm^2$ the area of the fan. Area of the fan covered by silk: To find the area covered by silk, consider the area not covered by silk as a sector of the circle with a radius of 5 cm. $A_{S'} = \frac{1}{3}(\pi)(5)^2$ $A_{S'} = 26.17\ sq\ cm^2$ the area of the part not covered by silk Then subtract the area not covered by the silk from the total area of the fan. $A_S = A_F - A_{S'}$ $= 654.17 - 26.17$ $= 628\ sq\ cm^2$ the area of the part covered by silk **Q3 a3:** Circumference of a circle is 2 x π x Radius The length of the arc is $(\frac{1}{3})(2)(\pi)(25) \approx 52.35$ cm Therefore, Jane needs approximately 53 cm of lace. **Q3 a4:** The radius of the fan from the rivet to the edge of the lace glued to the fan is $25 - 0.5 = 24.5$ cm $A_{L'} = \frac{1}{3}(3.14)(24.5)^2 = 628.26\ sq\ cm2$area of the fan from the rivet to the edge of the lace glued to the fan. To find the area with glued lace:

$A_L = A_F - A_{L'}$

$= 654.17 - 628.26$
$= 25.91\ sq\ cm2$

The four questions each represent a SOLO level. The correct answer for **Question 21 a1.** is at the *unistructural* level as it requires one simple operation resulting from dividing 360 by 120.

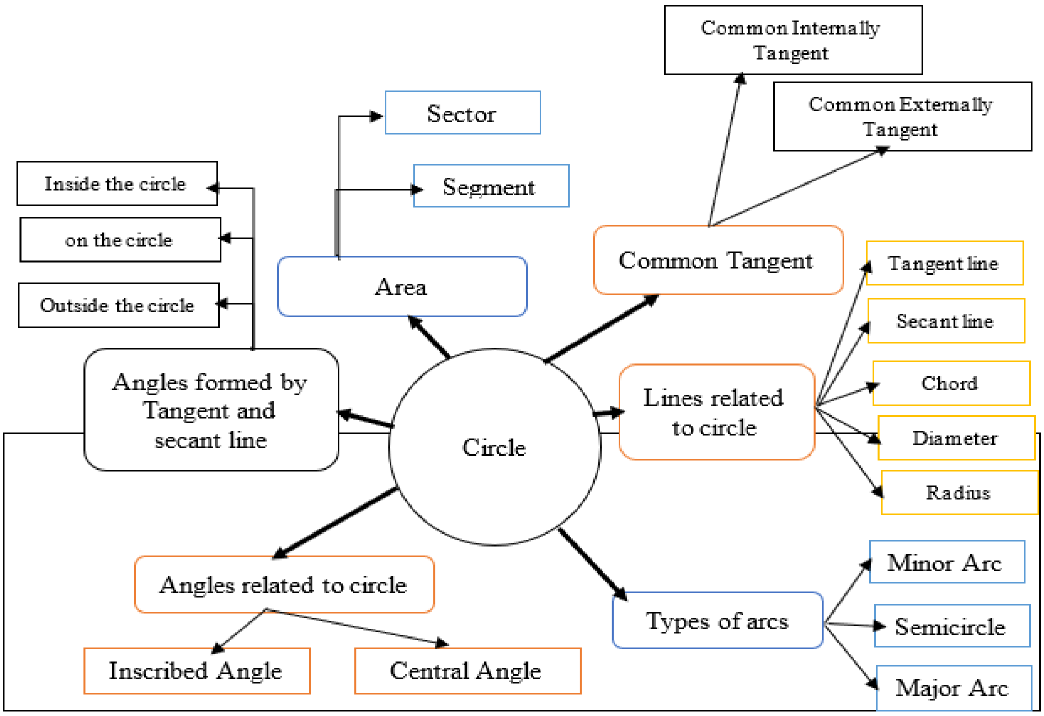
Question 3 a2., Question 3 a3. and **Question 3 a4.** all represent questions at a *multistructural* level as the respondent is expected to undertake a number of small calculations. There calculations for each question are in sequence. Some of the questions might appear more difficult for the respondent, but they are all examples of identifying given data and substituting these numbers into various known formulas associated with area or circumference of a circle.

None of the questions posed would be at the *relational* level as they do not require the need of some overview of the question requiring an overall response.

This question is important for teachers in helping understand the difference between lower-order and higher-order thinking. Nevertheless, it is important that respondents are able to complete successfully such questions as they represent the needed knowledge as a basis in order to move to higher-order thinking.

	QUESTION 4 Grade 10 Mathematics
004	**Q4 a1:** The error starts at **Step 2.** Since zero is included in the selection and has many placement restrictions. As a result, the solution cannot immediately employ the Fundamental Counting Principle. Instead, the solution must consider cases wherein zero is used and not used as an ending digit of the possible number. **Q4 a2:** Case I When an even number ends with zero. First digit: 5 choices Second digit: 4 choices Third digit: 3 choices Last digit: 1 choice Using the fundamental counting Principle $5 \times 4 \times 3 \times 1 = 60$ Case II: When an even number does not end with zero First digit: 4 choices since it will not start with zero Second digit: 4 choices Third digit: 3 choices Last digit: 2 choices Using the fundamental counting Principle $4 \times 4 \times 3 \times 2 = 96$ Adding Case I and Case II $60 + 96 = 156$ Possible PIN satisfying the conditions. **Q4 a3:** Teacher's judgment call NOTE: The reasons should focus on giving recognition to what the student has been able to achieve. How much was correct, not just counting steps. The number and seriousness of the mistakes. If one mistake is made and the following working using that mistake is correct, then the following work cannot be considered wrong mathematics. The only provisory here is if the mistake makes the question simpler. **Reflection:** **Question 4**. Expects respondents to find where the mistake has been made among the given answer/solution and explain why it was a mistake. Respondents are then asked to provide the correct answer to the question. Respondents are also asked to allocate marks for the wrong answer/s identified for an added 'fun' in the activity. This last part also adds some conceptual understanding and discussion. Identifying the answer in **Question 4 a1.** is most likely a *relational* response as it requires a holistic understanding of what the respondent has done but how it links to other issues.

	The second question, **Question 4 a2.,** requires a *multistructural* response as it requires a set of procedures performed in sequence, composed of a number of small calculations. The final question, **Question 4 a3.** can offer several levels of response. However, the highest level, *relational*, will require the respondent to first identify the mistake and the answer and find the balance of what has been achieved and what has not. This can be achieved by teasing out the magnitude of the error as compared to the amount of information and processes the respondent demonstrated that they understood.

	QUESTION 5 Grade 10 Mathematics
005	**Q5 a1:** Individual list, probably a few words such as: circumference, area, sector, centre, chord tangent, secant, arcs, segment, diameter, radius, etc **Note:** If respondents naturally work systematically in terms of Circle sub-topics for this question, then this is to be acknowledged when the class reports but not prompted at the beginning, see the next question. The idea of this first question is to see what individual respondents know in this area without support. These basic ideas are helping respondents move from first cycle (preliminary basic learning) to the second cycle. **Q5 a2:** Sub-topics identified by respondent might be: lines related to a circle; parts of a circle; angles on different parts of a circle; theorems about a circle; tangents to circles their properties; circle measurements; **Q5 a3:** **(i)** Agreed Group list Will include individual answers as well as ones that emerge as part of discussion **(ii)** Agreed Class list Will include individual answers as well as ones that emerge as part of discussion **Q5 a4:**  **Reflection:** The purpose of this activity is to use a series of lessons that has already been completed and ask the respondents to bring together the main ideas of the text/story/activity. The idea is to assist them organize these ideas in a logical way that

will help their understanding and present these ideas using **a grade-appropriate** and **age-relevant** concept map.

One great value for respondents with the development of a concept map is it provides a context in which respondents are expected to talk about mathematics, express their understandings or misunderstandings, and listen to and learn from, their peers as well as the teachers.

Most concept maps should have words on the arrows linking the main ideas. This way the different pathways present a 'story'.

Despite the aim of this activity to help respondents attain an overview to be answered at different levels, it is possible that respondents may give much lower levels than required for a complete response. Nevertheless, the overall activity is about presenting a finished product which is at the *relational* level for the concept that is being addressed at the knowledge level required. Great care must be taken not to push the concept map into areas where the respondents have not yet to encounter.

It is most likely that individual respondents will mainly be working at *unistructural* and *multistructural* SOLO levels. It will require encouragement and support to have a class end-product on what they have already achieved at the *relational* level.

In the case of **Question 5 a1.** a respondent might provide a number of single ideas about a single aspect of circles, such as names for parts of a circle. If this was the case, then the answer is in the first cycle. (NOTE: only giving (say) the name 'radius' is unistructural but in the first cycle.) For the respondents to enter the second cycle they need to start thinking in terms of Circle sub-topics. A response at the unistructural level in the second cycle will contain many of the words needed in a specific Circle sub-topic such as most of the names that describe different aspects of a circle. It will be significant by the end of the activity that the respondent can provide and talk about a number of relevant Circle sub-topics each made up of individual points if they are to be coded at the *multistructural* level see **Question 5 a2**. and **Question 5 a3.**

Question 5 a4. is at the *relational* level, second cycle, if it is bringing different sub-groups together in the same concept map. This would be a hoped-for result for capable Grade 10 respondents.

	QUESTION 6 Grade 10 Mathematics
006	**Q6 a1:** Because the question involves seating arrangement in a linear fashion, then order of the persons is important. Hence, we shall use permutation. There were 5 people involved including Alexander, so $n = 5$. Since all of them must be seated, then $r = 5$. The question asks for the permutation of 5 people taken 5 at a time. In symbols, $$P(n,r) = \frac{n!}{(n-r)!}$$ $$P(5,5) = \frac{5!}{(5-5)!} = \frac{5!}{0!} = 5! = 120 \text{ ways}$$ **Q6 a2:** The seating arrangement must be divided into 2 groups: for Alexander and his best friend in one group; and for the three other friends in the second group. There are 4 places on the bench that Alexander and his best friend could be placed on the long bench. This is at the beginning and end and in two places between each of the friends. Also, Alexander and his friend can sit side-by-side in 2 ways. The question now concerns how many ways can the three friends and Alexander-and-his-best-friend, i.e., 4 separate groups be seated, *P*(4,4). The total number of ways that all five people could they be seated together with the condition of Alexander's friend is given by $$P(2,2) \times P(4,4) = 2! \times 4! = 48 \; ways$$ **Q6 a3:** Here there becomes two pairs and one individual which we can consider as three separate elements. Therefore, there are 2 ways for each of the pairs and then the 3 separate elements to be allocated. *P*(2,2) x *P*(2,2) x *P*(3,3) = 24 ways **Note:** The problem could include some restrictions on how they could be seated. For example, consider a new question. **Q6 a4:** If all friends were to sit down with no restrictions there would be *P*(5,5) = 120 ways If Alexander and Alexander's friend who he had a bad time with him **did sit** together it would be on *P*(4,4) x *P*(2,2) = 48 ways If Alexander and Alexander's friend who he had a bad time with him **did NOT sit** together it would be on *120 – 48* = 72 ways **Q6 a5:** Class to discuss

Reflection:
Permutations require respondents to determine the number of ways some actions in an activity can be arranged. There are new symbols to be understood and ideas such as factorial illustrated by a '!' after a whole number. There is much basic work and practical activities to be undertaken to help the respondent establish the meaning of permutations. Most of this early work occurs in the first cycle of SOLO.

By the time respondents are at the *unistructural* level (in the second cycle) they can undertake simple tasks such as determining how many ways five people can be seated in five chairs. This is shown in **Question 6 a1.**

In **Question 6 a2**. the cognitive demands of the question increase by adding certain straightforward conditions. This question is at the *multistructural* level. It requires applying a similar idea to **Question 6 a1.** on more than one occasion. Sometimes, with some small additional observations that are performed sequentially.

In **Question 6 a3**., despite an increase in conceptual difficulty, the question part is still more of the same process as in **Question 6 a1**., but the idea needs to be applied a few times as in **Question 6 a2.** and is also at the *multistructural* level.

Question 6 a4. requires respondents to have a much deeper understanding of the process and the question as they need to hold a small number of important aspects together at the same time. It is about using the same basic formula but offers a more connected use and the process is not as predictable as the previous questions parts. This question is at the *relational* level.

Question 6 a5. Offers insight into respondent understanding. Producing a question plus the correct response can be found at all three levels. These levels will follow very closely the descriptions above, for example, a *unistructural* level will require a simple application of the permutation. The *relational* level will require the need for a considered response linking together different ideas or conditions.

Finally, it is important that respondents do not rely on the general counting principles established and apply them without thinking. The use of permutations can be a useful tool, but the respondent needs to be sure of the context in which it is to be used. Too often respondents apply a *unistructural* response in cases that require more deliberate consideration to establish whether and how a permutation might be used appropriately.

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